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ORIGINAL PAPER

Propagation of ion-acoustic localized mode excitations and their modulational instability analysis in electron–positron–ion plasmas

C Lavanya*

Department of Physics, Vivekanandha College of Arts and Sciences for Women (Autonomous), Elayampalayam, Tiruchengode, Namakkal, Tamilnadu 637205, India

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Abstract: The modulational instability of ion-acoustic waves (IAWs) in an unmagnetized multicomponent plasma is investigated, including a hot positrons, hot isothermal electrons and cold ions. Employing the reductive perturbation technique, the nonlinear Schrödinger equation (NLSE) is derived. The effects of positron concentration and temperature ratio of electron to positron significantly modify the modulational instability and its growth rate. Results show that increasing the strength of these parameters leads to localization of IAWs. Further, the exact traveling wave solutions are studied using a modified extended tanh function method. The relevance of theoretical results may be beneficent in understanding the localized electrostatic disturbances in space and astrophysical situations where electron–positron–ion plasmas are present.

Keywords: Reductive perturbation method; Ion acoustic solitary waves; Electron–positron–ion plasma; Soliton; Solitary wave solutions; Traveling wave solutions

1. Introduction

Nowadays, the study of electron–positron–ion (e–p–i) plasmas has been a growing interest because of its potential applications in astrophysical and laboratory observations [1, 2]. An e–p–i plasma is a fully ionized gas which consists of positrons and electrons. The occupancy of electron–positron (e–p) is also well known as pair plasmas. It possess indistinguishable masses and definite charge. The theoretical investigation of nonlinear structures in space, astrophysical and in laboratory multicomponent plasma has received a great deal of attention in plasma physics. The study of wave dynamics in e–p–i plasma is one of the most consequential aspects for plasma physicists, because of its extensive applications. The electron–positron plasmas are thought to be created naturally by pair production in high energy process happening in various astrophysical situation such as neutron stars [3], active galactic nuclei [4], pulsar magnetosphere [5], solar atmosphere [6], laboratory plasmas [7], semiconductor plasmas [8], relativistic jets that stream from the nuclei of quasars [9] and other inertial confinement fusion schemes [10] disclosed the phenomenon of nonlinear structures such as soliton, vortices, shocks and envelope holes etc. The e–p plasmas have also been generated in the laboratory by the use of modern positron trapping technique. The longer lifetime of the positron plays an important role in laboratory [11, 12] and astrophysical [13] plasmas due to the admixture of electrons, ions and positron. The study of wave motion in an e–p–i plasmas are completely distinct from those of two-component e–p plasma. According to that, the investigation of e–p–i plasmas are momentous to know the nature of linear and nonlinear properties of plasma waves [14–16]. Due to the plentiful existence of ions in numerous astrophysical plasmas, the e–p–i plasma has thought a great deal of theoretical consideration taking into account of positron concentration on the plasma dynamics [17–19].

The propagation of nonlinear waves especially solitary waves in e–p–i plasma is very attractive because of their theoretical aspect and also their applications. Solitons are single-pulse structures which are developed when nonlinearity balances with dispersion effects [20].

During the last few years, many researchers investigated the nonlinear propagation of waves in e–p–i plasmas. Berezhiani and Tskhakaya [21] comprehended the envelope solitons in e–p–i plasma by the propagation of electromagnetic waves. Nonlinear dynamics of ion-acoustic

*Corresponding author, E-mail: lavanya.is.c@gmail.com

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solitary waves in e–p–i plasmas have been furnished by Popel et al. [9]. They showed that the system supports positive potential ion-acoustic solitons with a reduction in amplitude under the impact of positron concentration. The nonlinear theory of IAWs in e–p–i plasmas have been analyzed by Dubinov and Sazonkin by employing pseudopotential technique [22]. Furthermore, the formation of ion-acoustic envelope solitary wave structures and modulational instability analysis in respect of parallel modulation of the carrier wave is investigated in magnetized and unmagnetized e–p–i plasma [23, 24]. The nonlinear propagation of ion-acoustic solitary wave for the weakly relativistic regime in an unmagnetized plasma composed of Boltzmann positrons, non-extensive electrons and relativistic ions have been theoretically and numerically explained by Hafez et al. [25].

The modulational instability (MI) is a natural phenomenon arising from the interplay of linear dispersion or diffraction and the nonlinear self-interaction of wave fields. MI is a primordial process concerted with the growth of perturbations on continuous wave background. MI has been increasing interest in fluid dynamics [26], nonlinear optics [27], Bose–Einstein condensates [28], plasma physics [29] and various fields. The study of modulational instability of the ion-acoustic waves (IAWs) is one of the noteworthy phenomena in plasma physics. Its basic mechanism involves the slow modulation of a monochromatic plane wave, resulting in the creation of localized pulses, which are described by nonlinear Schrödinger equation (NLSE). Tremendous attention has been paid to the investigation of the modulational instability of soliton in the case of nonlinear Schrödinger (NLS) equation due to their stable wave propagation. MI of monochromatic IAWs was first experimentally studied by Watanabe [30]. In recent years MI has been subjected to wide research, and its occurrence has been examined with respect to different wave modes in nonlinear and dispersive plasmas. The nonlinear, dispersive media has been a well-known phenomenon for the localization of wave energy. There are different theoretical studies have been examined on the MI of various wave modes [31]. Chawla et al. [32] reported the modulational instability of ion-acoustic waves consisting of warm adiabatic ions in a collisionless e–p–i plasma. The propagation of large amplitude IAWs in collisionless plasma consists of isothermal positrons, warm adiabatic ions and two temperature distribution of electrons were reported by Jain and Mishra [33]. Kourakis et al. [34] explained the nonlinear propagation of modulated electrostatic wave packets containing pair plasmas by employing a two-fluid plasma model. Tiwari et al. [35] reported the ion-acoustic dressed solitons in a three-component plasma containing hot electrons, cold ions and positrons. Nejoh [36] presented the large-amplitude ion-acoustic waves in an electron–positron–ion plasma under the influence of ion temperature. Jehan et al. [24] addressed that the oblique modulation of ion-acoustic waves and the formation of envelope soliton in a collisionless e–p–i plasma. It is found that the stability regions are changed by the presence of positron component for small angle of propagation with the direction of modulation.

Verheest and cattaert described the large amplitude soliton by the propagation of electromagnetic waves in e–p–i plasmas [37]. The propagation of nonlinear amplitude modulation of ion-acoustic wave in unmagnetized plasma in the presence of warm ion have been studied by Mahmood et al. [38]. The result shows that both positron density and ion temperature play a vital role in the formation of dark and bright envelope solitary wave structures. Gill et al. [39] studied the ion-acoustic solitary waves in three-component magnetized e–p–i plasma under the influence of positron concentration and q-nonextensive electrons.

More recently Ghosh et al. [40] investigated the dynamic behavior of ion acoustic waves in an unmagnetized plasma consisting of q-nonextensive electrons and positron by employing the bifurcation mechanism of planar dynamical systems through direct approach. They found that the existences of both solitary and periodic waves and their relevance to the physical parameters. Saha et al. [41] addressed the dynamic structures of ion acoustic waves in electron–positron–ion magnetoplasmas whose constituents are superthermal electrons and positrons. Applying the Hirota direct method to the Kadomtsev-Petviashvili (KP) equations the propagation of two-soliton and three-soliton waves are obtained. Maji et al. [42] investigated the oblique collision of two ion-acoustic waves (IAWs) in a three component plasma system composed of electrons, positrons and ions using Hirota direct method. The effects of the ratio of electron temperature to positron temperature and the ratio of the number density of positrons to that of electrons on the phase shifts are studied. Samanta et al. [43] studied the ion acoustic waves in two species plasma in the presence of external static magnetic field and kappa distributed electrons. By employing bifurcation theory of planar dynamical systems to the ZK equation. They found that the system supports solitary wave solutions and periodic travelling wave solutions. To the best of our knowledge, no theoretical study has been carried out on the examination of modulational instability and the exact traveling wave solutions for the system consisting of a hot positron, cold ions, hot isothermal electrons in an unmagnetized e–p–i plasmas by deriving the nonlinear Schrödinger equation. The intention of the present interpretation is to make a detailed study on the occurrence of modulational instability and growth rate in e–p–i plasmas including both the effects of positron concentration and the ratio of electron temperature to the positron.

The manuscript is structured as follows. In Sect. 2, the analytical model is addressed and by using the reductive perturbation technique a nonlinear Schrödinger equation is derived. The modulational instability analysis is pursued in Sect. 3. In Sect. 4, a traveling wave soliton solutions are obtained by employing modified extended tanh function method. Results and discussion are presented in Sect. 5 and 6 is devoted to conclusions.

2. The model equations

Here consider a multicomponent plasma system consisting of hot positrons, cold ions and hot isothermal electrons. The nonlinear dynamics of the ion-acoustic solitary waves in such a plasma system is governed by the following set of equations [44].

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - pn_p - (1-p)n, \tag{3}$$

$$n_e = \exp(\phi), \tag{4}$$

$$n_p = \exp(-\sigma\phi), \tag{5}$$

For small ϕ

$$n_e = 1 + \phi + \frac{\phi^2}{2} + \frac{\phi^3}{6} + \cdots, \tag{6}$$

$$n_p = 1 - \sigma\phi + \frac{\sigma^2\phi^2}{2} - \frac{\sigma^3\phi^3}{6} + \cdots \tag{7}$$

where $p = \frac{n_{p0}}{n_{e0}}$ and $\sigma = \frac{T_e}{T_p}$.

In the above equations, n and u represents the density and fluid velocity of the ion component, n_p and n_e are the density of positron and electron respectively. At equilibrium the densities of electron component, positron component and ion component are represented as n_{e0}, n_{p0} and n_0 respectively. ϕ denotes the electrostatic potential and p is the fractional concentration of positron with respect to electron in the equilibrium state. σ represents the temperature ratio of electron to positron. From Eq. (3), the electron and positron density distributions are considered to obey the Maxwell Boltzmann type. From Eqs. (1)–(5), velocity(u), potential(ϕ), time(t) and space variable(x) have been normalized by the ion sound velocity C_s, thermal potential $\frac{T_e}{e}$, inverse of the ion-plasma frequency ω_{pi}^{-1} and electron Debye length $\lambda_{De} = \left[\frac{\epsilon_0 T_e}{4\pi n_0 e^2}\right]^{\frac{1}{2}}$ respectively. The normalized equilibrium densities of ion density(n), electron density(n_e) and positron density (n_p) can be indicated as n_0, n_{e0} and n_{p0} respectively. T_e and T_p are the temperatures of electron and positron fluid respectively.

2.1. Derivation of the NLS equation

To investigate the modulational instability of ion acoustic solitary waves in electron–positron–ion plasma, we now follow the standard reductive perturbation theory to the model Eqs. (1)–(5) and construct a NLS equation [45, 46]. The independent variables are stretched as

$$\xi = \epsilon(x - v_g t), \qquad \tau = \epsilon^2 t, \tag{8}$$

where ϵ is a small parameter and v_g is the group velocity of the wave in the x direction. The dependent variables are expanded as

$$n = 1 + \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} n_l^{(n)}(\xi,\tau)\exp[i(kx-\omega t)l],$$

$$u = \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} u_l^{(n)}(\xi,\tau)\exp[i(kx-\omega t)l],$$

and

$$\phi = \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} \phi_l^{(n)}(\xi,\tau)\exp[i(kx-\omega t)l], \tag{9}$$

where n, u, ϕ satisfy the reality condition $A_{-l}^{(n)} = A_{-l}^{(n)^*}$ and the asterisk superscript denotes complex conjugate. k and ω represents the carrier wavenumber and frequency respectively. Substituting the expressions given in Eqs. (8) and (9 into Eqs. (1)–(5) and collecting the terms in the different powers of ϵ, we can establish the nth-order reduced equations. The derivative operators in the above equations can be represented as follows:

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \epsilon v_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau},$$
$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \epsilon \quad \frac{\partial}{\partial \xi}. \tag{10}$$

For the first order ($n = 1$) with $l = 1$ equations read

$$-i\omega n_1^{(1)} + iku_1^{(1)} = 0,$$
$$-i\omega u_1^{(1)} + ik\phi_1^{(1)} = 0,$$
$$-(k^2 + 1 + p\sigma)\phi_1^{(1)} + (1-p)n_1^{(1)} = 0. \tag{11}$$

From Eq. (11), we can get the first order quantities in terms of $\phi_1^{(1)}$ as

$$n_1^{(1)} = A_1\phi_1^{(1)}, \quad u_1^{(1)} = A_2\phi_1^{(1)} \tag{12}$$

where,

$$A_1 = \frac{k^2+1+p\sigma}{1-p}, A_2 = \frac{\omega}{k}\left[\frac{k^2+1+p\sigma}{1-p}\right].$$

The equations for $l = 1$ give rise to the following linear dispersion relation for the IAWs. The dispersion relation expresses the relation between the wave vector k and the frequency ω respectively.

$$\omega^2 = \frac{k^2(1-p)}{k^2+1+p\sigma}. \tag{13}$$

For the second-order ($n = 2$) reduced equations with $l = 1$, we obtain

$$\begin{aligned} n_1^{(2)} &= \frac{(k^2+1+p\sigma)\phi_1^{(2)}}{1-p} - \frac{2ik}{1-p}\frac{\partial \phi_1^{(1)}}{\partial \xi}, \\ u_1^{(2)} &= \frac{1}{ik}\left[i\omega\left[\frac{(k^2+1+p\sigma)\phi_1^{(2)}}{1-p} - \frac{2ik}{1-p}\frac{\partial \phi_1^{(1)}}{\partial \xi}\right] + \left(v_g A_1 - A_2\right)\frac{\partial \phi_1^{(1)}}{\partial \xi}\right], \\ \phi_1^{(2)} &= \frac{1}{\left[\omega^2(k^2+1+p\sigma)+k(p-1)\right]ik} \Big[-2\omega^2 k + \omega v_g A_1(p-1) \\ &\quad + \omega A_2(1-p) + v_g A_2 k(p-1) + k(1-p)\Big]\frac{\partial \phi_1^{(1)}}{\partial \xi}. \end{aligned} \tag{14}$$

Whereas the second order approximation $n = 2$ with the first harmonic $l = 1$, we deduce the following compatibility condition

$$v_g = \frac{d\omega}{dk} = \frac{k}{\omega}\left[\frac{1+p(\sigma-1-p\sigma)}{(k^2+1+p\sigma)^2}\right]. \tag{15}$$

The component of $l = 2$ for the second order ($n = 2$) reduced equations find the second order harmonic quantities.

$$n_2^{(2)} = B_1|\phi_1^{(1)}|^2, u_2^{(2)} = B_2|\phi_1^{(1)}|^2, \phi_2^{(2)} = B_3|\phi_1^{(1)}|^2. \tag{16}$$

The third order ($n = 3$) equations for the zeroth harmonics ($l = 0$) expressed as

$$n_0^{(2)} = B_4|\phi_1^{(1)}|^2, u_0^{(2)} = B_5|\phi_1^{(1)}|^2, \phi_0^{(2)} = B_6|\phi_1^{(1)}|^2, \tag{17}$$

where

$$\begin{aligned} B_1 &= \frac{(4k^2+1+p\sigma)B_3 + \frac{1}{2}(1-\sigma^2 p)}{p-1}, \\ B_2 &= \frac{1}{2\omega}\left[\frac{\omega^2}{k}\left(\frac{k^2+1+p\sigma}{1-p}\right)^2 + 2kB_3\right], \\ B_3 &= -\frac{1}{2}\left[\frac{\omega^2(p\sigma^2+p+k^4+2k^2(1+p\sigma)+2p\sigma)}{(p-1)(\omega^2(4k^2+1+p\sigma)-k^2p+k^2)}\right], \\ B_4 &= \frac{(\sigma^2 p-1)-(1+p\sigma)B_6}{p-1}, \\ B_5 &= \frac{v_g\left[-B_6(1+p\sigma)+(\sigma^2 p-1)\right]}{p-1} + \frac{2(k^2+1+p\sigma)^2\omega}{(1-p)^2 k}, \\ B_6 &= \frac{1}{p-1+v_g^2(1+p\sigma)k^2(p-1)} \Big[v_g k^2(p^2\sigma^2-p\sigma^2-p+1) + 4\omega k^3(1+p\sigma) \\ &\quad + 2\omega k + 4\omega kp\sigma + 2\omega kp^2\sigma^2 - 2\omega^2 k^4 \\ &\quad - 4\omega^2 k^2 p\sigma(1+p\sigma) \\ &\quad - 2\omega^2 - 4\omega^2 p\sigma - 2\omega^2 p^2\sigma^2\Big]. \end{aligned}$$

Finally, by using the above derived expressions for the third order ($n = 3$) and $l = 1$ component, we get the following nonlinear Schrödinger equation:

$$i\frac{\partial \phi}{\partial \tau} + R\frac{\partial^2 \phi}{\partial \xi^2} + S|\phi|^2\phi = 0, \tag{18}$$

where $\phi = \phi_1^{(1)}$ for simplicity. The dispersion coefficient R is given by

$$\begin{aligned} R &= \frac{1}{(k^2+1+p\sigma)k^3\omega^2(k^2+1+p\sigma)-k+p\sigma)} \Big[-4v_g k^3 p\sigma w^2 - 4v_g k^3\omega^2 \\ &\quad - 3v_g k\omega^2 - 2v_g k^3 p\sigma + 2\omega k^2 p\sigma + 2k^3\omega^3 \\ &\quad + 2v_g k^3 p - v_g k^5\omega^2 + 2v_g k^5 - 2\omega k^2 p^2\sigma \\ &\quad + 2v_g k^3 p^2\sigma + 2k^5\omega^3 - 2\omega k^4 p - 2\omega k^2 p \\ &\quad + 2v_g^2 k^2\omega - p^2\sigma^2\omega^3 k + 2\omega k^4 p - 2v_g k^6\omega^2 \\ &\quad 2v_g k^4\omega^2 - 3v_g k\omega^2 p^2\sigma^2 + 2v_g k^4\omega^2 p\sigma \\ &\quad + 4v_g^2 k^2\omega p\sigma + 4v_g^2 k^4\omega p\sigma + v_g k^2 p^2\sigma^2\omega^2 \\ &\quad + 2v_g^2 k^2\omega p^2\sigma^2 - 2p\sigma\omega^3 k - \omega^3 k - 2k^3\omega^3 \\ &\quad - k^5\omega^3 - 2v_g k^5 - \omega^3 k^4 + 2v_g k^2 p\sigma\omega^2 \\ &\quad - 6v_g k\omega^2 p\sigma + v_g k^2\omega^2 + 2v_g k^4\omega^2 + 2p\sigma\omega^3 \\ &\quad + \omega^3 - 2v_g k^4\omega^2 p\sigma + p^2\sigma^2\omega^3 - 2v_g k^3 \\ &\quad + 2\omega k^2 + v_g k^6\omega^2 + 4v_g^2 k^4\omega + 2v_g^2 k^6\omega\Big], \end{aligned}$$

and the nonlinear coefficient S is

$$S = -\frac{1}{2\omega(p-1)^2}\Big[2k\omega v_g p^2\sigma^2 - 2k\omega v_g p\sigma^2 + 6\omega^2 + 2k\omega v_g - 8k^2\omega^2 B_3 + 5\omega^2 p^2\sigma^2 - \omega^2 p\sigma^2 + 8\omega^2 k^2 p\sigma + 9\omega^2 k^2 + 2k\omega v_g B_6 - 2k\omega v_g B_6 p - 2k\omega v_g p - 2k\omega v_g p + 8k^2\omega^2 B_3 p + 2\omega^2 B_3 p^2\sigma - 2\omega^2 B_3 p\sigma - 2\omega^2 B_6 p^2\sigma + 2\omega^2 B_6 p\sigma + 9\omega^2 p\sigma + 2k^2 B_3 p^2 - 4k^2 B_3 p - 2k\omega v_g B_6 p^2\sigma + 2k\omega v_g B_6 p\sigma - 2\omega^2 B_3 + 2\omega^2 B_3 p + 2\omega^2 B_6 - 2\omega^2 B_6 p - \omega^2 p + 4\omega^2 k^4 + 2k^2 B_3\Big].$$

3. Modulational instability analysis to the NLS equation

In order to examine the modulational instability profile of the ion-acoustic wave in e–p–i plasma, we consider the plane wave solution for Eq. (18) in the form

$$\phi = \phi_0 exp[i(k\xi + \omega\tau)], \quad (19)$$

where ϕ_0 is the constant real amplitude. The real wave number (k) with (ω) as real frequency of the wave solution. Substituting the above solution in Eq. (18), we obtain the appropriate nonlinear (perturbation) dispersion relation.

$$\omega = S\phi_0^2 - Rk^2. \quad (20)$$

From Eq. (20), one can infer that the plane wave is nonlinear and the principle of superposition fails. In order to examine the MI of the carrier wave, we apply small perturbation of Eq. (19) which takes the form.

$$\phi = [\phi_0 + \delta\phi(\xi,\tau)]exp[i(k\xi + \omega\tau)], \quad (21)$$

where $\delta\phi$ is the small amplitude perturbation. Accordingly, the perturbed field grows exponentially, the steady state solution undergoes unstable. Substituting Eq. (21) into Eq. (18) and neglecting terms with higher orders of the perturbation amplitude, we get the linearized equation as.

$$i\delta\dot{\phi} - \omega\delta\phi + R[\delta\phi_{\xi\xi} + 2ik\delta\phi_\xi - k^2\delta\phi] + S[2\phi_0^2\delta\phi + \phi_0^2\delta\phi^*] = 0, \quad (22)$$

where the asterisk represents the complex conjugate. We seek for solutions of Eq. (22) in the form

$$\delta\phi = \eta_1 exp[i(Q\xi - \Omega\tau)] + \eta_2^* exp[-i(Q\xi - \Omega^*\tau)], \quad (23)$$

where Q and Ω are the modulational wave number and real perturbation frequency, η_1 and η_2^* are the complex constant amplitudes. Equation (23) represents a combination of progressive and regressive waves. Inserting Eq. (23) into Eq. (22) results in a set of two homogeneous equations, which are equivalent to the matrix equation

$$\begin{aligned}&\Big[\Omega - \omega + R(-Q^2 - 2kQ - k^2) + S(2\phi_0^2)\Big]\eta_1 + S\phi_0^2\eta_2 = 0,\\ &\Big[-\Omega - \omega + R(-Q^2 + 2kQ - k^2) + S(2\phi_0^2)\Big]\eta_2 + S\phi_0^2\eta_1 = 0,\end{aligned} \quad (24)$$

and the associated matrix can be expressed as,

$$\begin{pmatrix}\Omega + A & B\\ B & -\Omega + D\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}, \quad (25)$$

where

$$A = -\omega + R\Big[-Q^2 - 2kQ - k^2\Big] + S\Big[2\phi_0^2\Big],$$
$$B = S\phi_0^2,$$
$$D = -\omega + R\Big[-Q^2 + 2kQ - k^2\Big] + S\Big[2\phi_0^2\Big].$$

From Eq. (25) the following nonlinear dispersion relation for the amplitude modulation of ion acoustic wave is derived

$$\Omega = \frac{D - A}{2} \pm \frac{\sqrt{(A+D)^2 - 4B^2}}{2}. \quad (26)$$

Equation (26) possesses both the real and imaginary values of Ω. The real part of Ω represents a frequency shift relative to the uniform modes and the imaginary part of Ω determines the growth rate of the modulation of wave number Q. The stability of the nonlinear wave is determined by imaginary of Ω. It is apparent from this relation that if $Im\Omega > 0$ the wave will be modulational unstable otherwise it is stable. The stability nature of the wave is determined by the imaginary part of Ω. It is interesting to note from Eq. (26) that when $(A+D)^2 > 4B^2$, the wave is modulational stable due to the frequency of the wave is real for all the Q. On the other hand, the wave packet is modulational unstable when $(A + D)^2 < 4B^2$ because of the negative value for all the modulated wave number(Q) respectively. The MI gain $g(\Omega)$ is generally defined as $g(\Omega) = \pm Im(\Omega)$, and thus the instability growth rate is obtained as

$$g(\Omega) = \frac{\sqrt{4B^2 - (A+D)^2}}{2}. \quad (27)$$

where Im represents the imaginary part and the appearance of localized solitary structures exists only when the constant amplitude is undergoing unstable wavepacket. When $\Omega < 0$, the steady-state solution becomes unstable. Since the perturbation grows exponentially with the intensity given by the growth rate or MI gain. The study of linear stability analysis can predict the instability region in parameter space and shows qualitatively how the amplitude of a modulation sideband produces the onset of the instability.

4. Description of modified extended tanh function (METF) method

The study of exact traveling wave solutions has played a widespread role in nonlinear phenomena governing the nonlinear partial differential equations. It appears in various fields of science such as solid-state physics, fluid dynamics, plasma physics, optical fibers, chemical kinematics, geochemistry, fluid dynamics, chemical physics, elastic physics and so on. Solitons are the solutions of a spacious class of weakly nonlinear dispersive partial differential equations explaining the physical systems. It is arisen by the effect of nonlinear and dispersive terms in the medium. In recent years various substantial methods have been elucidated to inspect the exact solutions of nonlinear equations, such as Hirota direct method [47], tanh sech method [48], B*ä*cklund transformation method [49], homogeneous balance method [50], F-expansion method [51], (G'/G) expansion method [52], Jacobi elliptic function method [53] were employed to find the solution for dispersive and dissipative problems. Several authors put much effort to seek exact soliton solutions to nonlinear partial differential equations. Tanh method is one of the most powerful direct methods for establishing the wave solution of nonlinear partial differential equations (NLPDE). This method was firstly reported by Malfliet [54]. Later, Fan [55] has developed the extended tanh function method and found out the traveling wave solutions that cannot be obtained by the tanh function method. Recently, using the modified extended tanh method the new exact traveling wave solutions have developed by El-Wakil et al. [56]. To illustrate the basic concepts of the modified extended tanh-function method (METF), we consider a following nonlinear PDEs as follows

$$F(u, u_\xi, u_\tau, u_{\xi\xi}, \ldots) = 0, \tag{28}$$

when we look for its traveling wave solutions, the first step is to introduce the wave transformation $u(\xi, \tau) = u(\gamma)$, $\gamma = \xi + c\tau$ or $\gamma = \xi - c\tau$ and to change the Eq. (28) to an ordinary differential equation (ODE) of the form:

$$F(u, u_\gamma, u_{\gamma\gamma}, \ldots) = 0. \tag{29}$$

The next crucial step is that we propose the following series expansion for a solution of Eq. (29)

$$u(\gamma) = a_0 + \sum_{i=1}^{N}(a_i\varphi^i + \sum_{i=1}^{N} b_i\varphi^{-i}), \tag{30}$$

and

$$\frac{d\varphi}{d\gamma} = b + \varphi^2, \tag{31}$$

where b is a parameter to be resolved and the positive integer N can be found by balancing the highest order linear term with the nonlinear terms in (29). Substituting Eqs. (30–31) into Eq. (29) and then setting zero to all the coefficients of φ^i, we can obtain a system of algebraic equations, from which the constants a_i, b_i, b, c (where $i = 0, \ldots N$) are obtained explicitly. Fortunately, the Riccati equation admits several types of solutions: (a) If $b < 0$

$$\varphi = \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\gamma), \\ -\sqrt{-b}\coth(\sqrt{-b}\gamma) \end{cases}$$

(b) If $b = 0$:

$$\varphi = \frac{-1}{\gamma}, \tag{32}$$

(c) If $b > 0$

$$\varphi = \begin{cases} \sqrt{b}\tan(\sqrt{b}\gamma), \\ -\sqrt{-b}\cot(\sqrt{b}\gamma) \end{cases}$$

4.1. Application of description method to NLS equation

To study the exact traveling wave solutions of the nonlinear Schr*ö*dinger equation given in Eq. (18), we consider a plane wave transformation in this form:

$$\phi(\xi, \tau) = P(\gamma) + iM(\gamma). \tag{33}$$

Applying the transformation $\phi(\xi, \tau) = \phi(\gamma)$, $\gamma = \xi - c\tau$ to Eq. (18), we obtain the following differential equation

$$\begin{aligned} &-icP_\gamma - M_\gamma + RP_{\gamma\gamma} + iRM_{\gamma\gamma} + SP^3(\gamma) \\ &+ iSP^2(\gamma)M(\gamma) + iSM^3(\gamma) \\ &+ iSM^2(\gamma)P(\gamma) = 0. \end{aligned} \tag{34}$$

Separating real and imaginary parts of the equation

$$M_\gamma + RP_{\gamma\gamma} + SP^3(\gamma) + SM^2(\gamma)P(\gamma) = 0, \tag{35}$$

$$-cP_\gamma + RM_{\gamma\gamma} + SP^2(\gamma)M(\gamma) + SM^3(\gamma) = 0. \tag{36}$$

The solution can be expressed as

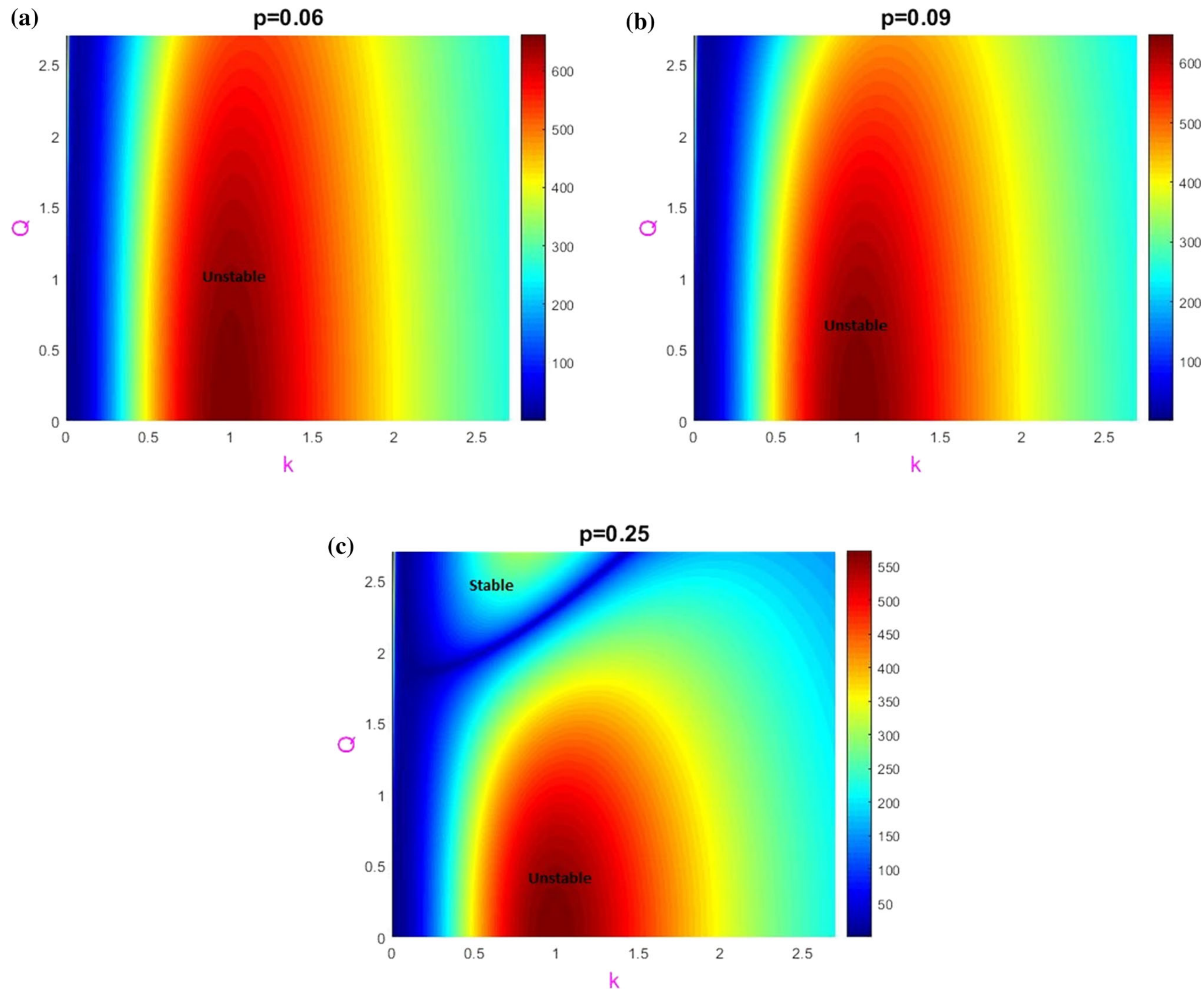

Fig. 1 Snapshots of stability/instability region in the (K, Q) plane with the choices of parameters $\phi_0 = 1.8$, $\omega = 0.04$. Along with $\sigma = 0.5$

$$P(\gamma) = a_0 + \sum_{i=1}^{N} (a_i \varphi^i + b_i \varphi^{-i}), \tag{37}$$

$$M(\gamma) = b_0 + \sum_{j=1}^{N} (c_j \varphi^j + d_j \varphi^{-j}). \tag{38}$$

Substituting $\varphi'(\gamma) = b + \varphi^2$ and balancing the highest order linear term with the nonlinear term we get the value of N as 1. So, the solution takes the form:

$$P(\gamma) = a_0 + a_1 \varphi + b_1 \varphi^{-1}. \tag{39}$$

$$M(\gamma) = b_0 + c_1 \varphi + d_1 \varphi^{-1}. \tag{40}$$

Upon substituting Eqs. (39–40) in the ordinary differential Eqs. (35–36) will yield a system of algebraic equations with respect to a_i, b_i, c_i, d_i and b. The system of equations is futher solved by using symbolic computation and we obtain

$$\begin{aligned} a_1 &= \frac{1}{18} \left[\frac{6b_0^2 S R a_0^2 - 2b_0^4 S R - b_0^2 c^2 - 3a_0^2 c^2}{c b_0 a_0^2 S} \right], \\ b_1 &= \frac{6 S a_0^2 b_0 c}{2 b_0^2 S R + c^2}, \\ c_1 &= -\left[\frac{2 b_0^2 S R + c^2}{3 a_0 S c} \right], d_1 = 0, b = \frac{-6 b_0^2 S^2 a_0^2}{2 b_0^2 S R + c^2}. \end{aligned} \tag{41}$$

Then by inserting Eq. (41) in Eqs. (39–40) and the solution recasts:

$$\begin{aligned} P(\gamma) &= a_0 + a_1 \left[-\sqrt{-b} tanh \left[\sqrt{-b} (\xi - c\tau) \right] \right] \\ &\quad + b_1 \left[-\sqrt{-b} tanh \left[\sqrt{-b} (\xi - c\tau) \right] \right]^{-1}, \\ M(\gamma) &= b_0 + c_1 \left[-\sqrt{-b} tanh \left[\sqrt{-b} (\xi - c\tau) \right] \right]. \end{aligned} \tag{42}$$

Hence solutions of Eq. (33) turn out to be

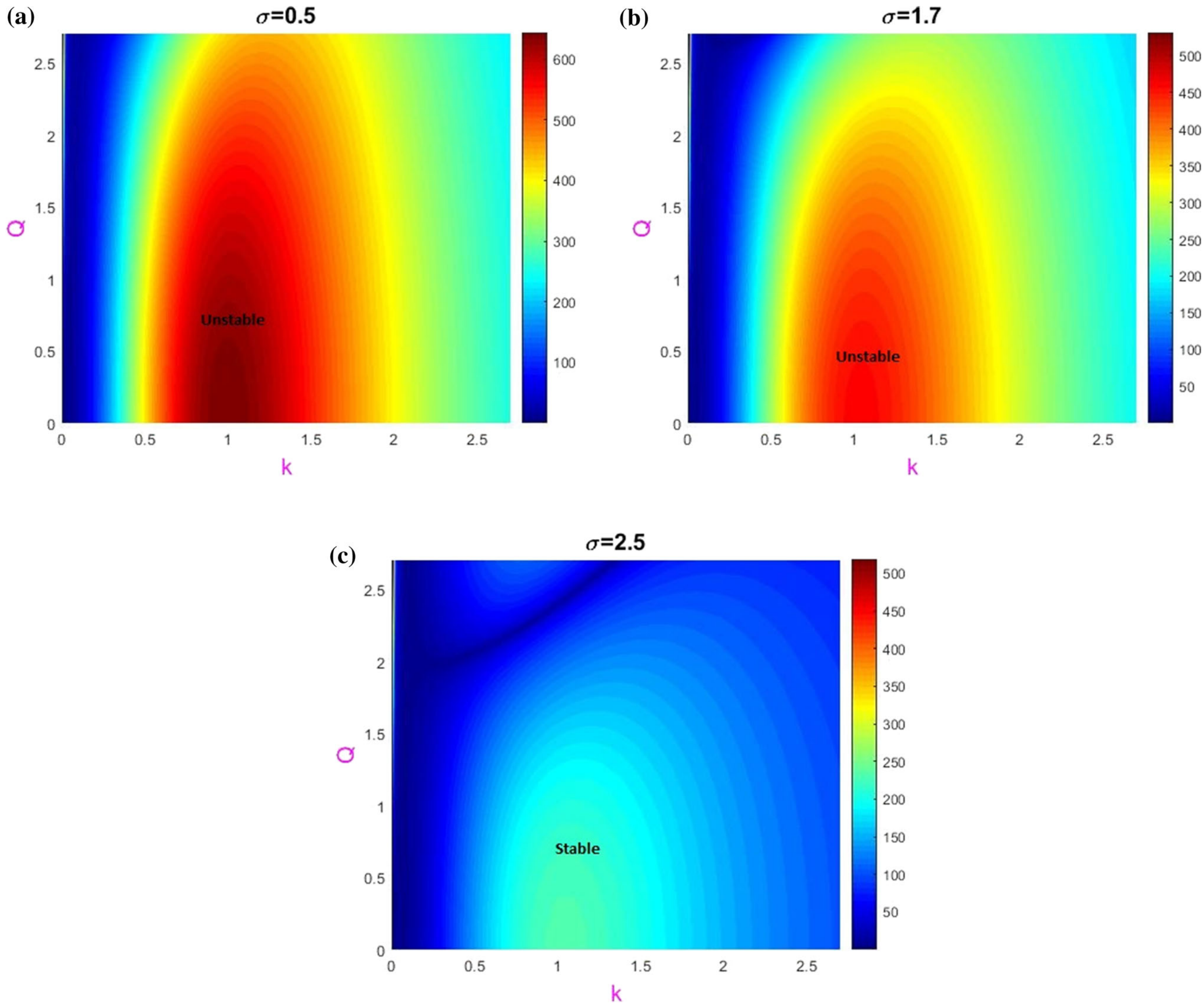

Fig. 2 Snapshots of stability/instability region in the (K, Q) plane with the choices of parameters $\phi_0 = 1.8$, $\omega = 0.04$. Along with $p = 0.1$

$$\begin{aligned}\phi = a_0 &+ \frac{6b_0^2SRa_0^2 - 2b_0^4SR - b_0^2c^2 - 3a_0^2c^2}{18cb_0a_0^2S} \\ &\Big[-\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}} \\ &\tanh\Big[\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}}(\xi - c\tau)\Big]\Big] \\ &+ \frac{6Sa_0^2b_0c}{2b_0^2SR + c^2}\Big[-\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}}\times \\ &\tanh\Big[\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}}(\xi - c\tau)\Big]\Big]^{-1} \\ &+ i\Big[b_0 - \frac{2b_0^2SR + c^2}{3a_0Sc}\Big[-\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}} \\ &\tanh\Big[\sqrt{\frac{6b_0^2S^2a_0^2}{2b_0^2SR + c^2}}(\xi - c\tau)\Big]\Big]\Big]. \end{aligned} \tag{43}$$

where Eq. (43) represents the traveling wave solutions of Eq. (18) which describes the propagation of ion acoustic soliton.

5. Results and discussion

In this model of plasma, I have studied the modulational instability analysis of ion-acoustic solitary waves in an electron–positron–ion plasma considering the effects of positron concentration (p) and the ratio of electron temperature to positron(σ) respectively. Figures 1 and 2 show the stability/instability zone of the growth rate of the wavenumber (k, Q) plane for different values of relevant plasma parameters for the solution of Eq. (27). From Fig. 1(a-c) we can obviously note that, for any fixed arbitrary

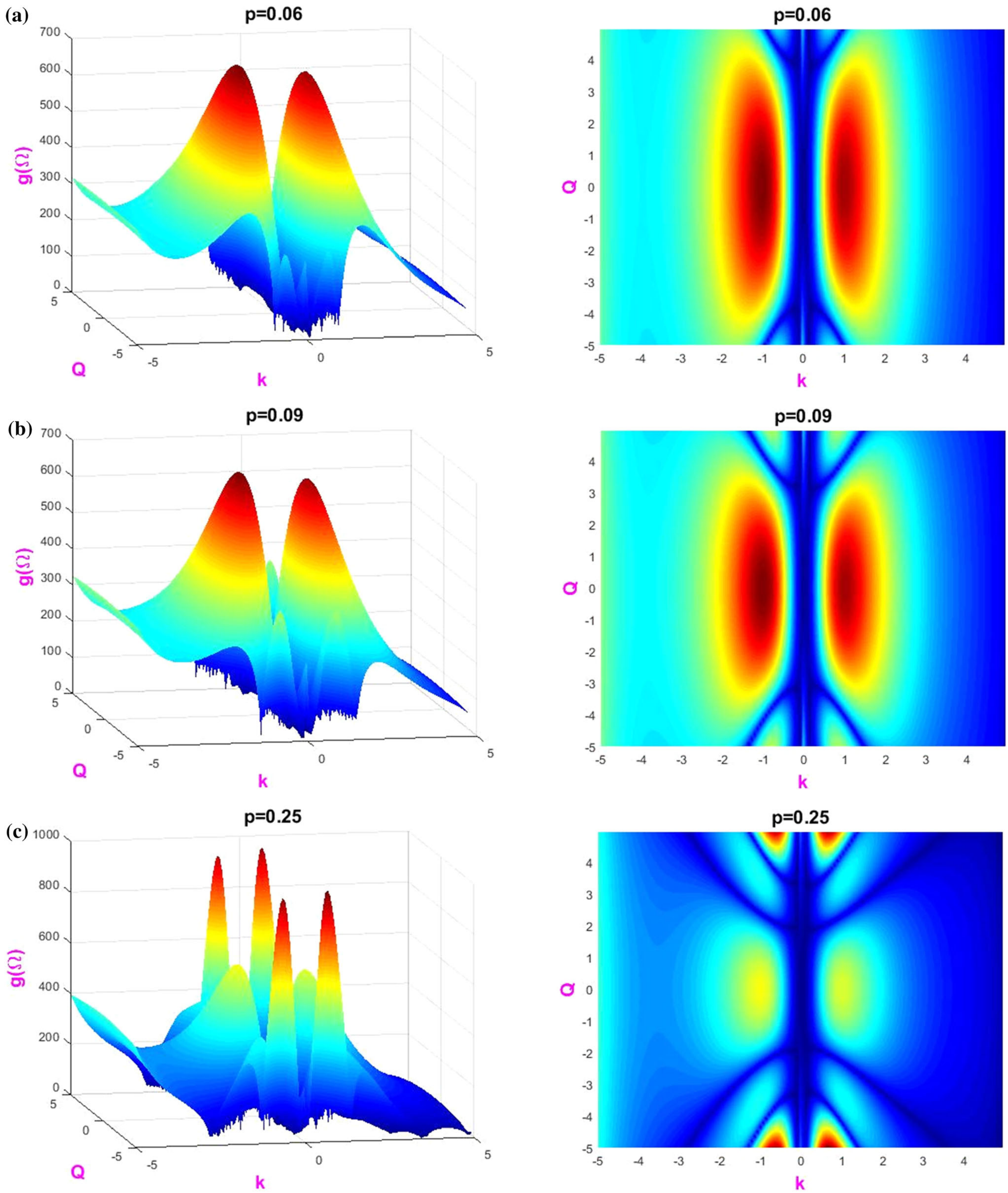

Fig. 3 The modulational instability gain of Eq. (27) for the parametric values of $\phi_0 = 1.8$, $\omega = 0.04$ when $\sigma = 0.5$

values of $\phi_0 = 1.8$, $\omega = 0.004$ and $\sigma = 0.5$, the instability growth rate is shown to suppress with increasing positron concentration. Figure 2 demonstrates that the effect of σ on the stability/instability region of the growth rate of IAWs when the other parameters are fixed. Thus, the ratio of electron temperature to positron (σ) plays a crucial role to

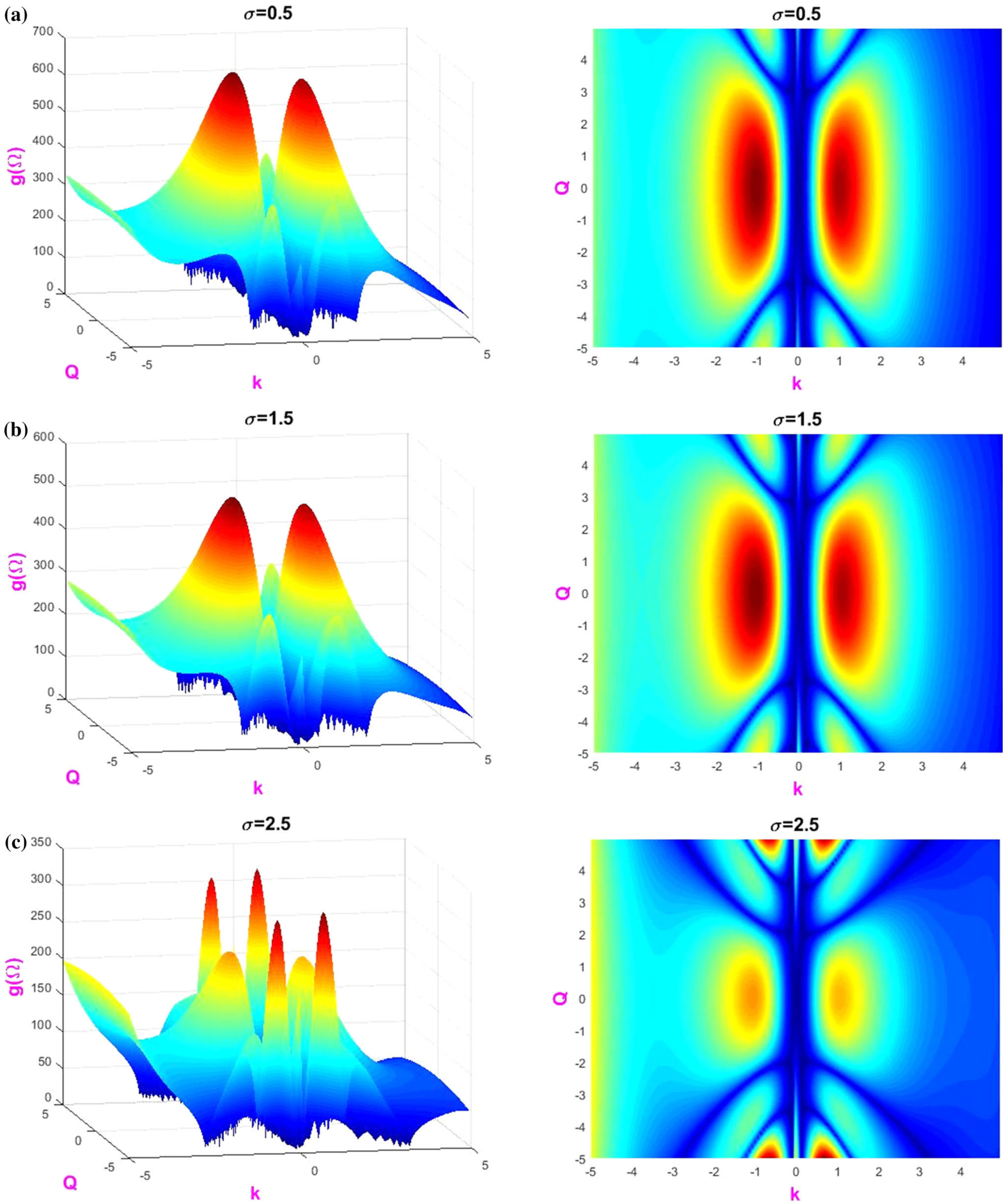

Fig. 4 The modulational instability gain of Eq. (27) for the parametric values of $\phi_0 = 1.8$, $\omega = 0.04$ when $p = 0.1$

change the stability of the wave packets. The group dispersive coefficient (R) and the nonlinear coefficient (S) depend on a number of physical parameters such as positron concentration and the ratio of electron temperature to positron. Thus, these physical parameters are expected to significantly influence the stability characteristics of the

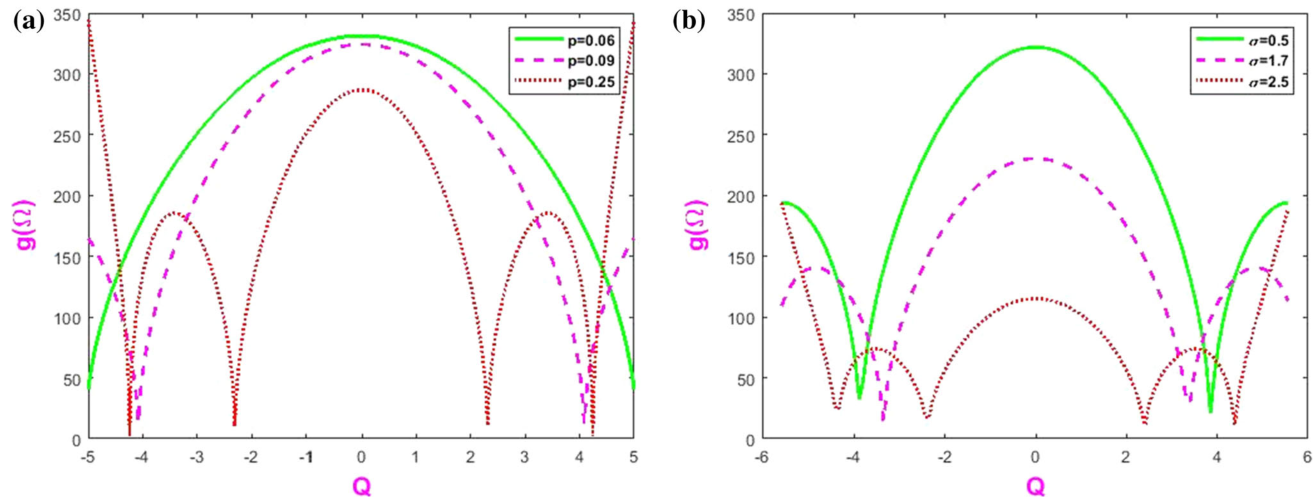

Fig. 5 Snapshots of stability/instability region in the (K, Q) plane with the choices of parameters $\phi_0 = 1.8$, $\omega = 0.04$. Along with (**a**) $\sigma = 0.5$ and (**b**) $p = 0.1$

modulated ion-acoustic wave. Therefore, it is important to study the dependence of the stability profile of the modulated ion-acoustic waves on various physical parameters. From plot (a) in Fig. 3, it is found that for any fixed arbitrary values of $\phi_0 = 1.8$, $\omega = 0.004$ and $\sigma = 0.5$, the localized soliton excitation is obtained. Further increasing the positron concentration the unstable zone is suppressed as depicted in Fig. 3(b, c). From this plot, we can observe that the presence of positron concentration plays a marvelous role to generates a stable wave. As is widely known, the driving force of the ion-acoustic wave is offered by the ion inertia and the increasing positron concentration (p) implies the depopulation of ions. The corresponding 2d plot is also inspected in Fig. 3(a–c). As obvious from the plot, the amplitude of the solitary wave decreases with increasing the value of p. This situation indicates that the soliton energy decreases with an increasing p. From Fig. 4, for a definite value of $p = 0.1$ and keeping the other parameters constant and for $\sigma = 0.5$ leads to the evolution of localized solitary wave structures. Further increasing the controlling parameter σ from 1.7 to 2.5, the amplitude of the soliton is abruptly decreased. From Fig. 4, it is evident that for a selected set of parameters the region of the instability is decreased. The corresponding 2d plot is exploited in Fig. 4(a–c). The interesting finding from this investigation is that the soliton energy is decreased due to the effects of both the physical parameters p and σ. To obtain more information about the modulational instability, we plot in Fig. 5(a, b) the modulational instability growth rate, under the influence of p and σ. Figure 5(a) portrays that the growth rate $g(\Omega)$ as functions of Q with different values of $(p) =$ 0.06 (solid line), 0.09 (dashed line) and 0.25 (dotted line). It is obvious that as the value of p increases the growth rate is

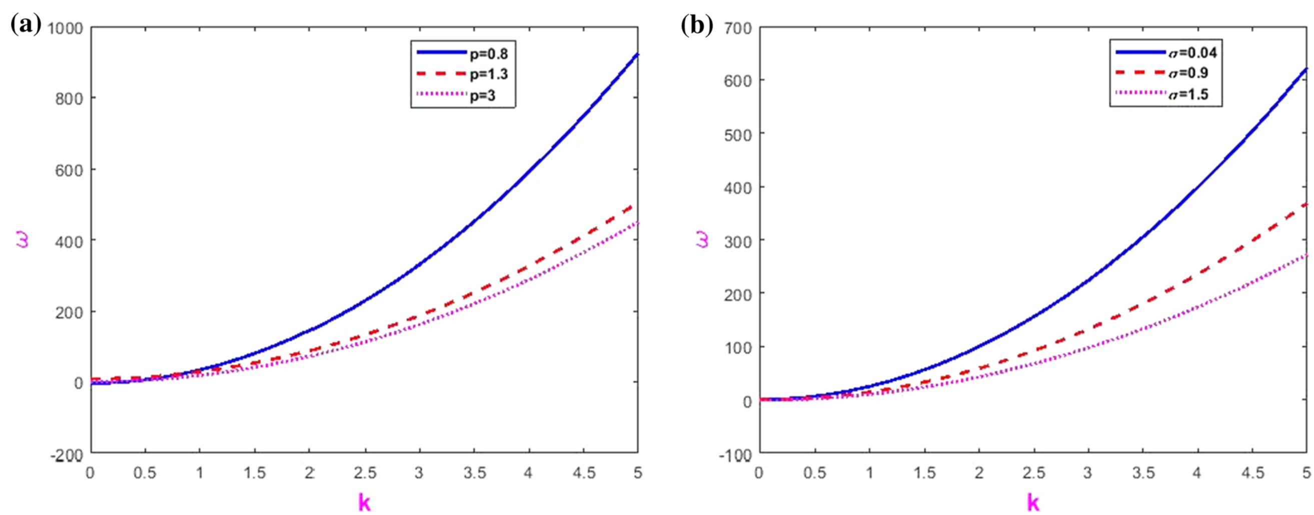

Fig. 6 Variation of the frequency ω against the wave number (k) for different values of p with (**a**) $\sigma = 0.09$ and different values of σ with (**b**) $p = 0.5$

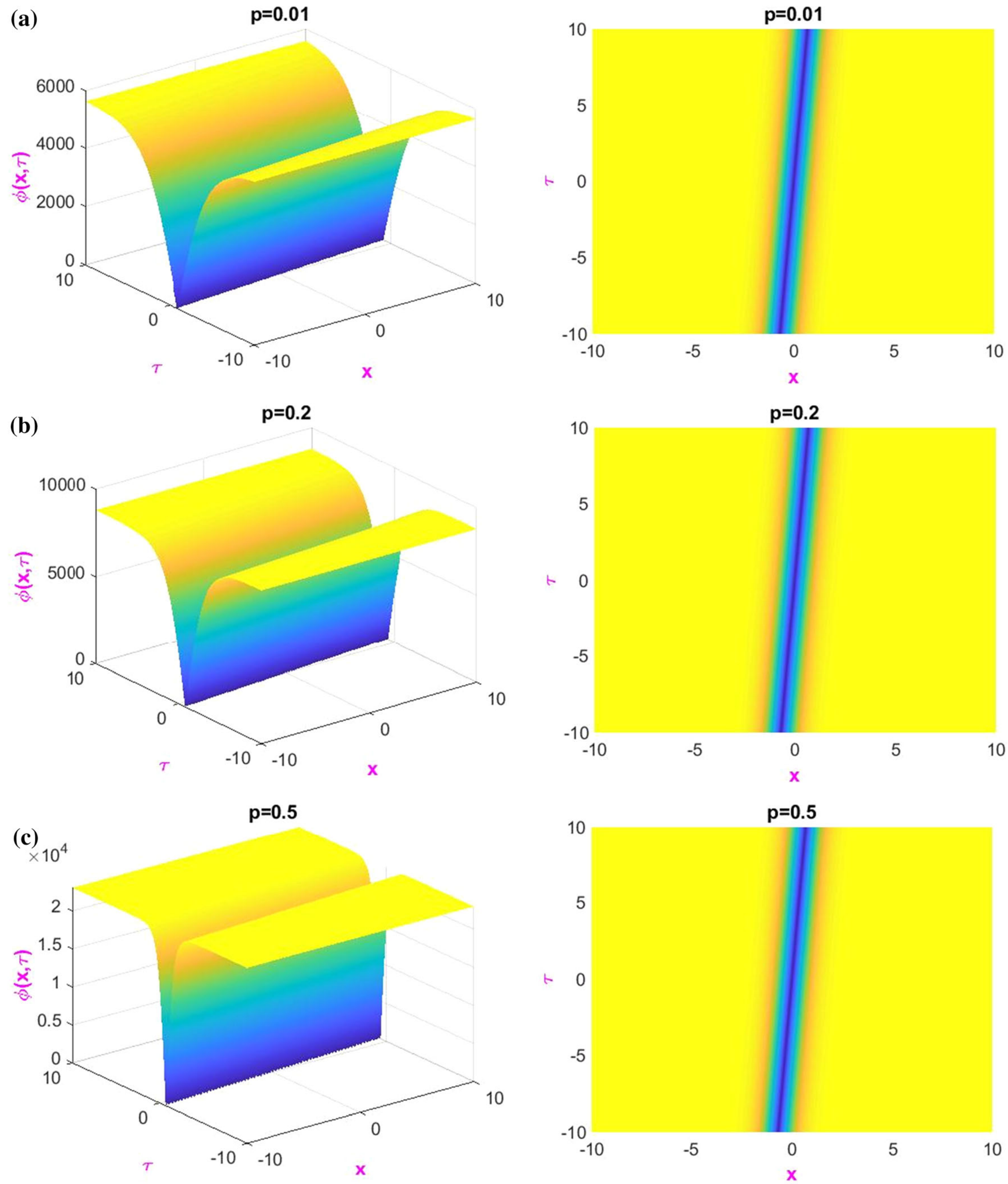

Fig. 7 Snapshots of anti-soliton excitations and its contour plots (**a–c**) for Eq. (43) with respect to the positron concentration (p). Other parameters are taken as $\omega = 0.8$, $k = 0.03$, $c = 0.05$, $a_0 = 0.8$, $b_0 = 0.1$ and $\sigma = 0.05$

gradual decreases. Apparently, as p increases, both amplitude and width are observed to decrease. In Fig. 5(b), depicts that the variation of $g(\Omega)$ with respect to wavenumber Q for various values of $\sigma = 0.5$ (solid line), 1.7 (dashed line) and 2.5 (dotted line) with constant values of $\phi_0 = 1.8$, $\omega = 0.004$ and $\sigma = 0.5$. This result shows that an increase of σ the growth rate of instability is diminished. It is found that the amplitude and width of the soliton decrease with an increase of σ. Physically, the increasing of the (σ) lead to dissipate the energy from the system and reduce the nonlinearity that makes the IAWs amplitude shorter. So the positron concentration (p) and the ratio of electron temperature to positron σ can be recognized to suppress of the MI growth rate of our considered plasma model.

The dispersion relation for IAWs (Eq. 20) to propagate in plasma medium is presented in Fig. 6. Figure 6(a) depicts that variation of frequency (ω) versus wave number (k) for different values of p. It is clear from Fig. 6(a) that an

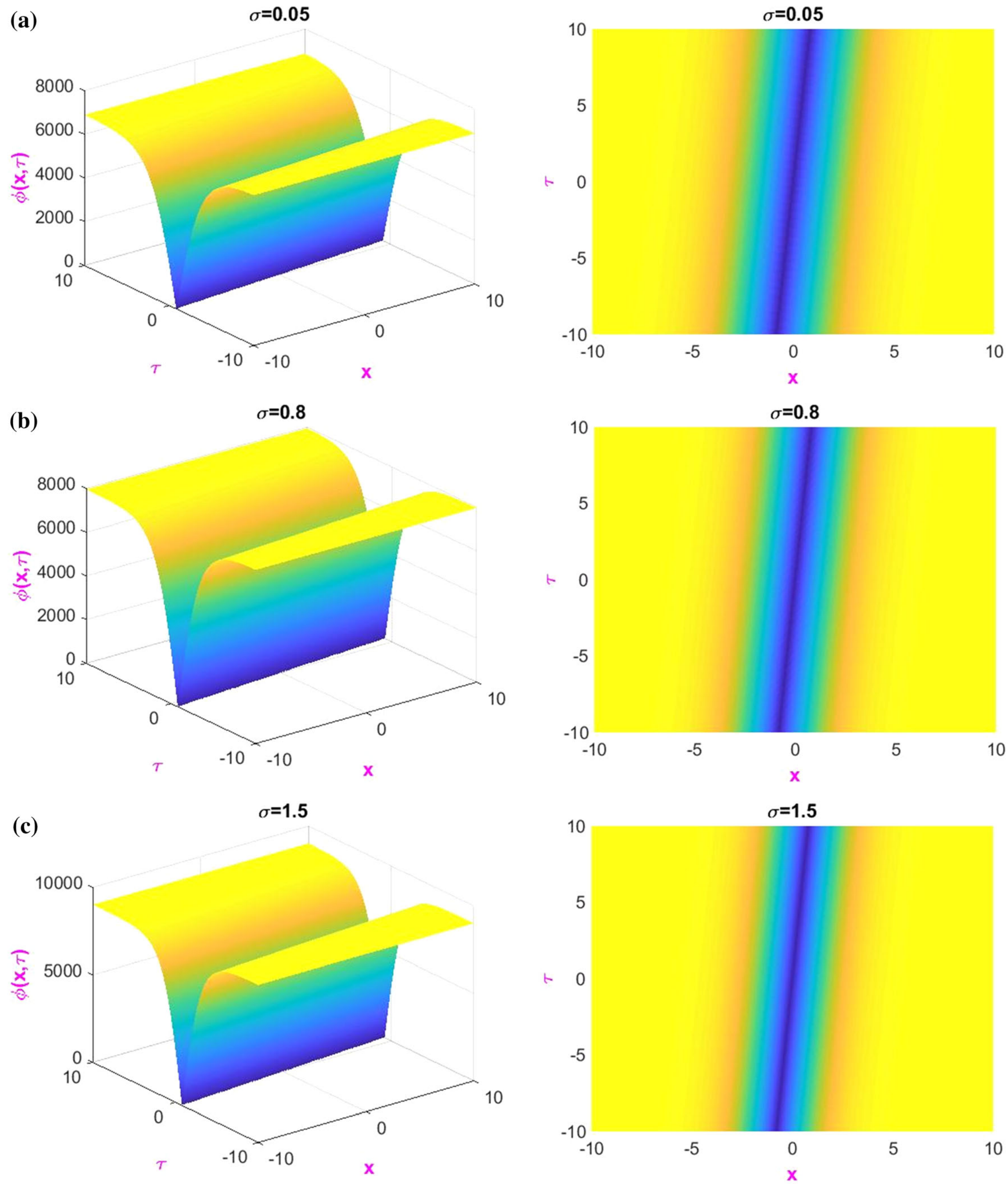

Fig. 8 Snapshots of antisoliton excitations and its contour plots (**a–c**) for Eq. (43) with respect to the ratio of electron temperature to positron (σ). Other parameters are taken as $\omega = 0.8$, $k = 0.03$, $c = 0.05$, $a_0 = 0.8$, $b_0 = 0.1$ and $p = 0.1$

increase in the value of p would lead to decrease in the value of the ω. On the other hand, higher values of the σ result in lower frequency values at constant values of $p = 0.5$ is shown in Fig. 6(b). Figure 7 indicates the profile of the exact traveling wave solutions of Eq. (43) by choosing the parametric values $\omega = 0.8$, $k = 0.03$, $c = 0.05$, $a_0 = 0.8$, $b_0 = 0.1$ and $\sigma = 0.05$. Upon increasing the positron concentration the solution takes the form of anti-soliton structure. Also, the enhancement in the positron concentration causes some fluctuations in the amplitude of the soliton which is shown in Fig. 7(b, c). A similar effect is witnessed in Fig. 8 for different values of electron temperature to a positron (σ). Figure 8(a) portrays that for $\sigma = 0.05$, the solution exhibits antisoliton structures. Further increasing the value of (σ) the amplitude of the soliton is changed as depicted in Fig. 8(b, c). This seems that the increase in positron concentration (p) and the ratio of electron temperature to a positron (σ) change the amplitude of the IAWs system.

6. Conclusions

In this paper, I have considered a multicomponent plasma consisting of hot positrons, cold ion and hot isothermal electrons. Using the reductive perturbation technique, the NLS equation has been derived. The modulational instability of ion-acoustic waves and localized excitations in electron–positron–ion plasmas have been studied under the influence of positron concentration and the ratio of electron temperature to positron. Furthermore, the growth rate of instability has been analyzed. It is observed that the stability of these solitary structures strongly depend on both the parameters p and σ. According to modified extended tanh-function method (METF) the exact traveling wave solutions are obtained. The effects of the physical parameters inaugurates the antisoliton excitations. It is also manifested that the significance of plasma parameters affect the amplitude and width of the soliton. From this analysis we can conclude that the present results will be meaningful in studying the features of nonlinear excitations in space and astrophysical plasmas in the presence of electron–positron–ion plasmas.

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