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INTUITIONISTIC FUZZY G″′-CLOSED SETS

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ABSTRACT

In this paper we introduce intuitionistic fuzzy g″′-closed sets and intuitionistic fuzzy g″′- open sets. The relations between intuitionistic fuzzy g″′-closed sets and other intuitionistic fuzzy generalized closed sets are given.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [17] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of generalized closed sets in topological spaces was introduced by Levine [8]. In this paper we introduce intuitionistic fuzzy g″′-closed sets and intuitionistic fuzzy g″′- open sets. The relations between intuitionistic fuzzy g″′-closed sets and other generalizations of intuitionistic fuzzy closed sets are given.

2. PRELIMINARIES

Definition 2.1 [1]

An intuitionistic fuzzy set (IFS in short) A in X can be described in the form $A = \{\langle, \mu A(x), \nu A(x)\rangle | x \in X\}$ and X be a non empty set where the function $\mu A : X \to [0, 1]$ is called the membership function and $\mu A(x)$ denotes the degree to which $x \in A$ and the function $\nu A : X \to [0, 1]$ is called the non-membership function and $\nu A(x)$ denotes the degree to which $x /\in A$ and $0 \leq \mu A(x) + \nu A(x) \leq 1$ for each $x \in X$. Denote IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout the paper, X denotes a non empty set.

Definition 2.2 [1]

Let A and B be any two IFSs of the form A={⟨x, μA(x), νA(x)⟩ | x ∈ X} and B={⟨x, μB(x), νB(x)⟩ | x ∈ X}. Then

(1). A ⊆ B if and only if μA(x) ≤ μB(x) and νA(x) ≥ νB(x) for all x ∈ X,

(2). A = B if and only if A ⊆ B and B ⊆ A,

(3). A_c = {⟨x, νA(x), μA(x)⟩ | x ∈ X},

(4). A ∩ B = {⟨x, μA(x) ∧ μB(x), νA(x) ∨ νB(x)⟩ | x ∈ X},

(5). A ∪ B = {⟨x, μA(x) ∨ μB(x), νA(x) ∧ νB(x)⟩ | x ∈ X}.

Definition 2.3 [1]

The intuitionistic fuzzy sets $0_\sim$ = {⟨x, 0, 1⟩ | x ∈ X} and $1_\sim$ = {⟨x, 1, 0⟩ | x ∈ X} are called the empty set and the whole set of X respectively.

Definition 2.4 [1]

Let A and B be any two IFSs of the form A = {⟨x, μA(x), νA(x)⟩ | x ∈ X} and B = {⟨x, μB(x), νB(x)⟩ | x ∈ X}. Then

(1). A⊆B and A⊆C ⇒ A ⊆ B∩C,

(2). A⊆C and B⊆C ⇒ A∪B ⊆ C,

(3). A⊆B and B⊆C ⇒ A ⊆ C,

(4). $(A\cup B)_c = A_c \cap B_c$ and $(A\cap B)_c = A_c \cup B_c$

(5). $((A)_c)_c = A$,

(6). $(1_\sim)_c = 0_\sim$ and $(0_\sim)_c = 1_\sim$

Definition 2.5 [3].

An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A_c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6 [3]

Let (X, τ) be an IFTS and A = {⟨x, μA(x), νA(x)⟩ | x∈X} be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:

int(A) = ∪ {G | G is an IFOS in X and G ⊆ A},

cl(A) = ∩ {K | K is an IFCS in X and A ⊆ K}.

Proposition 2.7 [3]

For any IFSs A and B in (X, τ), we have

(1). int(A) ⊆ A,

(2). A ⊆ cl(A),

(3). A is an IFCS in X ⇔ cl(A) = A

(4). A is an IFOS in X ⇔ int(A) = A,

(5). A ⊆ B ⇒ int(A) ⊆ int(B) and cl(A) ⊆ cl(B),

(6). int(int(A)) = int(A),

(7). cl(cl(A)) = cl(A),

(8). cl(A∪B) = cl(A) ∪ cl(B),

(9). int(A∩B) = int(A) ∩ int(B).

Proposition 2.8 [3]

For any IFS A in (X, τ), we have

(1). int($0_\sim$)=$0_\sim$ and cl($0_\sim$)=$0_\sim$,

(2). int($1_\sim$)=$1_\sim$ and cl($1_\sim$)=$1_\sim$,

(3). $(\text{int}(A))_c = \text{cl}(A_c)$,

(4). $(\text{cl}(A))_c = \text{int}(A_c)$.

Proposition 2.9 [3]

If A is an IFCS in (X, τ) then cl(A) = A and if A is an IFOS in (X, τ) then int(A) = A. Then arbitrary union of IFCSs is an IFCS in (X, τ).

Definition 2.10

An IFS A in an IFTS (X, τ) is said to be an

(1). intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) ⊆ A, [5]

(2). intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) ⊆ A, [2]

(3). intuitionistic fuzzy semi pre closed set (IFSPCS in short) if int(cl(int(A))) ⊆ A. [16]

Definition 2.11

An IFS A in an IFTS (X, τ) is said to be an

(1). intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(int(A))),[5]

(2). intuitionistic fuzzy semi open set (IFSOS in short) if A ⊆ cl(int(A)),[4]

(3). intuitionistic fuzzy semi pre open set (IFSPOS in short) if A ⊆ cl(int(cl(A))). [16]

Remark 2.12 [7]

We have the following implications.

IFCS → IFαCS → IFSCS → IFSPCS

None of the above implications are reversible.

Definition 2.13 [12]

Let A be an IFS in an IFTS (X, τ). Then the α-interior of A (αint(A) in short) and the α-closure of A (αcl(A) in short) are defined as

αint(A) = ∪ {G | G is an IFαOS in (X, τ) and G ⊆ A},

αcl(A) = ∩ {K | K is an IFαCS in (X, τ) and A ⊆ K}.

s-int(A), s-cl(A), s-pint(A) and s-pcl(A) are similarly defined. For any IFS A in (X, τ), we have αcl(Ac) = (αint(A))c and αint(Ac)=(αcl(A))c

Remark 2.14 [12]

Let A be an IFS in an IFTS (X, τ). Then

(1). αcl(A) = A ∪ cl(int(cl(A))),

(2). αint(A) = A ∩ int(cl(int(A))).

Definition 2.15

An IFS A in (X, τ) is said to be an

(1). intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ), [14]

(2). intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ),[11]

(3). intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ),[13]

(4). intuitionistic fuzzy α generalized closed set (IFαGCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ), [12]

(5). intuitionistic fuzzy α generalized semi closed set (IFαGSCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ), [6]

(6). intuitionistic fuzzy ω closed set (IFωCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ), [13]

(7). intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ). [10]

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Remark 2.16 [13]

(1). Every IFOS is an IFSGOS,

(2). Every IFSOS is an IFSGOS.

Definition 2.17 [15]

Two IFSs A and B are said to be q-coincident (AqB in short) if and only if there exists an element x ∈ X such that μA(x) > νB(x) or νA(x) < μB(x). For any two IFS A and B of (X, τ), $A\bar{q}B$ if and only if A ⊆ B_c

3. INTUITIONISTIC FUZZY G″′ -CLOSED SETS

Example 3.7 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.2,0.3), (0.7,0.6)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A= ⟨x, (0.3,0.4), (0.6,0.5)⟩. Here IFG″′C(X) = {0~,G^c,1~}. Therefore A is not an IFG″′CS in (X,τ). And we have spcl(A) = A. Therefore A is an IFGSPCS in (X,τ).

Theorem 3.8 Every IFG‴CS is an IFωCS.

Proof. Let A⊆U where U is an IFSOS in (X, τ). Since every IFSOS is an IFGSOS and since A is an IFG‴CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). we have cl(A) ⊆U, whenever A⊆U and U is an IFSOS in (X, τ). Therefore A is an IFωCS in (X,τ). Hence every IFG‴CS is an IFωCS.

The converse of the part is need not be true as seen from the following Example.

Example 3.9 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.3,0.4), (0.6,0.5)⟩. We have μ_G(a)=0.3, μ_G(b)=0.4, ν_G(a)=0.6 and ν_G(b)=0.5. Consider an IFS A= ⟨x, (0.7,0.6), (0.2,0.3)⟩. Here IFG‴C(X) = {0~,G^c,1~}. Therefore A is not an IFG‴CS in (X,τ). And here IFSO(X) = {0~,G,A,G^c,1~} where G⊂A⊂G^c. Therefore A is an IFωCS in (X,τ).

Theorem 3.10 Every IFG‴CS is an IFGCS.

Proof. Let A⊆U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG‴CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). we have cl(A) ⊆U, whenever A⊆U and U is an IFOS in (X, τ). Therefore A is an IFGCS in (X,τ). Hence every IFG‴CS is an IFGCS.

The converse of the part is need not be true as seen from the following Example.

Example 3.11 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.6,0.5), (0.3,0.4)⟩. We have μ_G(a)=0.6, μ_G(b)=0.5, ν_G(a)=0.3 and ν_G(b)=0.4. Consider an IFS A= ⟨x, (0.7,0.6), (0.2,0.3)⟩. Here IFG‴C(X) = {0~,A,G^c,1~}where 0~⊂A⊂G^c. Therefore A is not an IFG‴CS in (X,τ). And here U=1~ is the only IFOS which contains A. Therefore A is an IFGCS in (X,τ).

Theorem 3.12 Every IFG‴CS is an IFαGCS.

Proof. Consider A⊆U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG‴CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). we have cl(A) ⊆U, whenever A⊆U and U is an IFOS in (X, τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U. Therefore A is an IFαGCS in (X,τ). Hence every IFG‴CS is an IFαGCS.

The converse of the part is need not be true as seen from the following Example.

Example 3.13 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.6,0.7), (0.3,0.2)⟩. We have μ_G(a)=0.6, μ_G(b)=0.7, ν_G(a)=0.3 and ν_G(b)=0.2. Consider an IFS A= ⟨x, (0.6,0.7), (0.3,0.2)⟩. Here IFG‴C(X) = {0~,A,G^c,1~}where 0~⊂A⊂G^c. Therefore A is not an IFG‴CS in (X,τ). And here U=1~ is the only IFOS which contains A. Therefore A is an IFαGCS in (X,τ).

Theorem 3.14 Every IFG″′CS is an IFGSCS.

Proof Let A⊆U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG″′CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). we have cl(A) ⊆U, whenever A⊆U and U is an IFOS in (X, τ). Since scl(A) ⊆ cl(A), we have scl(A) ⊆ U. Therefore A is an IFGSCS in (X,τ). Hence every IFG″′CS is an IFGSCS.

The converse of the part is need not be true as seen from the following Example.

Example 3.15 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.2,0.3), (0.7,0.6)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A= ⟨x, (0.6,0.7), (0.3,0.2)⟩. Here IFG″′C(X) = {0~,G^c,1~}. Therefore A is not an IFG″′CS in (X,τ). And here U=1~ is the only IFOS which contains A. Therefore A is an IFGSCS in (X,τ).

Definition 3.16 An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy g_s″′-closed set(IFG_s″′CS in short) if scl(A)⊆Uwhenever A⊆UandUis an IFGSOS in (X, τ).

The complement of an intuitionistic fuzzy g_s″′-closed set is called an intuitionistic fuzzy g_s″′-openset(IFG_s″′OS in short).

Theorem 3.17 Every IFG″′CS is an IFG_s″′CS.

Proof. Let A⊆U where U is an IFGSOS in (X, τ). Since A is an IFG″′CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). Since scl(A) ⊆ cl(A), we have scl(A) ⊆ U. Therefore A is an IFG_s″′CS in (X,τ). Hence every IFG″′CS is an IFG_s″′CS.

The converse of the part is need not be true as seen from the following Example.

Example 3.18 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.2,0.3), (0.7,0.6)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A= ⟨x, (0.3,0.4), (0.6,0.5)⟩. Here IFG″′C(X) = {0~,G^c,1~}. Therefore A is not an IFG″′CS in (X,τ). And here IFSC(X) = {0~,G,A,G^c,1~} where G⊂A⊂G^c. Therefore A is an IFG_s″′CS in (X,τ).

Theorem 3.19 Every IFG″′CS is an IFαGSCS.

Proof Let A⊆U where U is an IFSOS in (X, τ). Since every IFSOS is an IFGSOS and since A is an IFG″′CS in (X,τ), we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ).We have cl(A) ⊆U,wheneverA⊆Uand U is an IFSOS in (X, τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U. Therefore A is an IFαGSCS in (X,τ). Hence every IFG″′CS is an IFαGSCS.

The converse of the part is not be true as seen from the following Example.

Example 3.20 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.4,0.3), (0.5,0.6)⟩. We have μ_G(a)=0.4, μ_G(b)=0.3, ν_G(a)=0.5 and ν_G(b)=0.6. Consider an IFS A= ⟨x, (0.6,0.7), (0.3,0.2)⟩. Here

IFG″′C(X) = {$0_{\sim}$ G ^{c},$1_{\sim}$}.Therefore A is not an IFG″′CS in (X,τ). And here IFSO(X)={$0_{\sim}$ G,A,G^{c},$1_{\sim}$}where G⊂A⊂G^{c}. Therefore A is an IFαGSCS in (X,τ).

Definition 3.21 An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy g_{α}″′-closed set(IFG_{α}″′CS in short) if αcl(A)⊆Uwhenever A⊆UandUis an IFGSOS in (X, τ).

The complement of an intuitionistic fuzzy g_{α}″′-closed set is called an intuitionistic fuzzy g_{α}″′-open set(IFG_{α}″′OS in short).

Theorem 3.22 Every IFG″′CS is an IFG_{α}″′CS.

Proof. Let A be an IFG″′CS in (X,τ). Then we have cl(A) ⊆U, whenever A⊆U and U is an IFGSOS in (X,τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U, whenever A⊆U and U is an IFGSOS in (X, τ). Therefore A is an IFG_{α}″′CS in (X,τ). Hence every IFG″′CS is an IFG_{α}″′CS.

The converse of nt part is need not be true as seen from the following Example.

Example 3.23 Let X={a,b}. Let τ ={$0_{\sim}$,G,$1_{\sim}$} be an IFT on X, where G=⟨x, (0.3,0.6), (0.6,0.3)⟩. We have μ_G(a)=0.3, μ_G(b)=0.6, ν_G(a)=0.6 and ν_G(b)=0.3. Consider an IFS A=⟨x, (0.2,0.3), (0.7,0.6)⟩.Then A is an IFG_{α}″′CS but not an IFG″′CS in (X,τ).

Remark 3.24 IFαCS and IFG″′CS are independent.

Example 3.25 Let X={a,b}. Let τ ={$0_{\sim}$,G,$1_{\sim}$} be an IFT on X, where G=⟨x, (0.2,0.3), (0.8,0.7)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.8 and ν_G(b)=0.7. Consider an IFS A=⟨x, (0.1,0.4), (0.9,0.6)⟩. Then A is an IFαCS but not an IFG″′CS in (X,τ).

Example 3.26 Let X={a,b}. Let τ ={$0_{\sim}$,G_1,G_2,$1_{\sim}$} be an IFT on X, where G_1=⟨x, (0.3,0.2), (0.6,0.7)⟩ and G_2= ⟨x, (0.3,0.4), (0.6,0.5)⟩. We have μ_{G1}(a)=0.3, μ_{G1}(b)=0.2, ν_{G1}(a)=0.6, ν_{G1}(b)=0.7, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider an IFS A=⟨x, (0.65,0.75), (0.25,0.15)⟩. Then A is an IFG″′CS but not an IFαCS in (X,τ).

Remark 3.27 IFSCS and IFG″′CS are independent.

Example 3.28 Let X={a,b}. Let τ ={$0_{\sim}$,G,$1_{\sim}$} be an IFT on X, where G=⟨x, (0.3,0.4), (0.6,0.5)⟩. We have μ_G(a)=0.3, μ_G(b)=0.4, ν_G(a)=0.6 and ν_G(b)=0.5. Consider an IFS A=⟨x, (0.4,0.5), (0.5,0.4)⟩. Then A is an IFSCS but not an IFG″′CS in (X,τ).

Example 3.29 Let X={a,b}. Let τ ={$0_{\sim}$,G_1,G_2,$1_{\sim}$} be an IFT on X, where G_1=⟨x, (0.2,0.3), (0.7,0.6)⟩ and G_2= ⟨x, (0.3,0.4), (0.6,0.5)⟩. We have μ_{G1}(a)=0.2, μ_{G1}(b)=0.3, ν_{G1}(a)=0.7, ν_{G1}(b)=0.6, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider an IFS A=⟨x, (0.8,0.7), (0.1,0.2)⟩. Then A is an IFG″′CS but not an IFSCS in (X,τ).

Theorem 3.30 If A and B are IFG″′CSs in an IFTS (X,τ), then A U B is also an IFG″′CS in (X,τ).

Proof If A U B ⊆ G where G is IFGSOS, then A ⊆ G and B ⊆ G. Since A and B are IFG″′CSs, cl(A) ⊆ G and cl(B) ⊆ G and hence cl(A) U cl(B) = cl(AUB) ⊆ G. Thus AUB is an IFG″′CS in (X,τ).

Remark 3.31 The intersection of two IFG″′CSs in an IFTS (X,τ) need not be an IFG″′CS in (X,τ).

Example 3.32 Let X={a,b}. Let τ ={$0_\sim$,G_1,G_2,$1_\sim$} be an IFT on X, where G_1=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$ and G_2= $\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have μ_{G1}(a)=0.2, μ_{G1}(b)=0.3, ν_{G1}(a)=0.7, ν_{G1}(b)=0.6, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider the two IFSs A= $\langle x,(0.1,0.7),(0.8,0.2)\rangle$ and B=$\langle x,(0.8,0.2),(0.1,0.7)\rangle$. Then A and B are IFG″′CSs.But A ∩ B = $\langle x,(0.1,0.2),(0.8,0.7)\rangle$ is not an IFG″′CS in (X,τ).

Theorem 3.33 If A is an IFG″′CSs in an IFTS (X,τ) and A ⊆ B ⊆ cl(A), then B is an IFG″′CS in (X,τ).

Proof If B ⊆ U where U is an IFGSOS in (X,τ). Since A ⊆ B and A⊆ U. Since A is an IFG″′CS in (X,τ), cl(A) ⊆ U. Since B⊆cl(A), cl(B) ⊆ cl(A)⊆ U. Therefore B is an IFG″′CS in (X,τ).

Theorem 3.34 Let A be an IFS in an IFTS (X,τ). Then A is an IFG″′CS if and only if A$\bar{q}$F implies cl(A)$\bar{q}$F for every IFGSCS F in (X,τ).

Proof Necessary Part: Let F be an IFGSCS in (X,τ) and Let A$\bar{q}$F. Then A ⊆ F^c, where F^c is an IFGSOS in (X,τ). Therefore by hypothesis cl(A) ⊆ F^c. Hence cl(A)$\bar{q}$F.

Sufficient Part: Let F be an IFGSCS in (X,τ) and Let A be an IFS in (X,τ). BY hypothesis, A$\bar{q}$F implies cl(A)$\bar{q}$F. Then cl(A) ⊆ F^c whenever A ⊆ F^c and F^c is an IFGSOS in (X,τ). Hence A is an IFG″′CS in (X,τ).

Theorem 3.35 Let (X,τ) be an IFTS. Then IFC(X) = IFG″′C(X) if every IFS in (X,τ) is an IFGSOS in X, where IFC(X) denotes the collection of IFCSc of an IFTS (X,τ).

Proof Suppose that every IFS in (X,τ) is an IFGSOS in X. Let A∈ IFG″′C(X). Then cl(A) ⊆ U whenever A ⊆ U and U is an IFGSOS in X.Since every IFS is an IFGSOS,A is also an IFGSOS and A ⊆ A. Therefore cl(A) ⊆ A. Hence cl(A) = A. Therefore A∈ IFC(X). Hence IFG″′C(X) ⊆ IFC(X) →(1). Let A∈IFC(X). Then by Theorem 3.4, A∈ IFG″′C(X). Hence IFC(X) ⊆ IFG″′C(X) →(2). From (1) and (2), we have IFC(X) = IFG″′C(X).

Proposition 3.36 If A is an IFGSOS and IFG″′CS in an IFTS (X,τ), then A is an IFCS in (X,τ).

Proof Since A is an IFGSOS and IFG″′CS, cl(A) ⊆ A. Hence A is an IFCS in (X,τ).

4. INTUITIONISTIC FUZZY g''' -open sets

In this section we introduce intuitionistic fuzzy g″′-open sets and study some of its properties.

Definition 4.1 An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy g″′-open set(IFG″′OS in short) if A^c is an intuitionistic fuzzy g″′-closed set in (X,τ).

The collection of all intuitionistic fuzzy g″′-open sets in X is denoted by IFG″′O(X).

Example 4.2 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.2,0.3), (0.7,0.6)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A=⟨x, (0.2,0.3), (0.7,0.6)⟩. Then by Example 1.2.2, A^c is an IFG″′CS in (X,τ). Hence A is an IFG″′OS in (X,τ).

Example 4.3 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.2,0.3), (0.7,0.6)⟩. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A=⟨x, (0.6,0.5), (0.3,0.4)⟩. Then by Example 4.3, A^c is not an IFG″′CS in (X,τ). Hence A is not an IFG″′OS in (X,τ).

Theorem 4.4 An IFS A in an IFTS (X,τ) is IFG″′OS if and only if F⊆int (A) whenever F⊆A and F^c is an IFGSOS.

Proof Necessary part: Let A be an IFG″′OS in (X,τ). Let F^c be an IFGSOS such that F⊆A . Then $A^c \subseteq F^c$ Where $A^c$is an IFG″′CS. Hence cl(A^c) ⊆ F^c. This implies (int(A))$^c \subseteq F^c$. Thus we have F ⊆ int (A) whenever F⊆A and F^c is an IFGSOS.

Sufficient part: Let F ⊆ int (A) whenever F⊆A and F^c is an IFGSOS in (X,τ). This implies (int(A))$^c \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is an IFGSOS. That is cl(A^c) ⊆ F^c whenever $A^c \subseteq F^c$ and F^c is an IFGSOS. Therefore A^c is an IFG″′CS. Hence A is an IFG″′OS in (X,τ).

Theorem 4.5 Every IFOS is an IFG″′OS .

Proof. Let A be an IFOSin (X,τ). Therefore A^c is an IFCS in (X,τ). Then by theorem 4.4,A^c is an IFG″′CS in(X,τ). Therefore A is an IFG″′OS in(X,τ).

The converse of the statement is need need not be true as seen from the following example.

Example 4.6 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=⟨x, (0.7,0.6), (0.2,0.3)⟩. We have μ_G(a)=0.7, μ_G(b)=0.6, ν_G(a)=0.2 and ν_G(b)=0.3. Consider an IFS A=⟨x, (0.8,0.7), (0.1,0.2)⟩. Then by Example, A^c is an IFG″′CS but not an IFCS in (X,τ). Hence A is an IFG″′OS but not an IFOS in (X,τ).

Theorem 4.7 Every IFG″′OS is an IFGSPOS.

Proof . Let A be an IFG″′OS in(X,τ). ThereforeA^c is an IFG″′CS in(X,τ). Then by theorem 1.2.6, A^c is an IFGSPCS in (X,τ). Therefore A is anIFGSPOS in (X,τ).

The converse of theorem 4.7 need not be true as seen from the following example.

Example 4.8 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$. We have $\mu_G(a)$=0.2, $\mu_G(b)$=0.3, $\nu_G(a)$=0.7 and $\nu_G(b)$=0.6. Consider an IFS A=$\langle x,(0.6,0.5),(0.3,0.24)\rangle$. Then by Example 4.7, A^c is an IFGSPCS but not an IFG″′CS in (X,τ). Hence A is an IFGSPOS but not an IFG″′OS in (X,τ).

Theorem 4.9 Every IFG″′OS is an IFωOS.

Proof. Let A be anIFG″′OS in (X,τ). Therefore A^c is an IFG″′CS in (X,τ). Then by Theorem, A^c is an IFωCS in(X,τ). Therefore A is an IFωOS in(X,τ).

The converse of theorem 4.9 need not be true as seen from the following Example.

Example 4.10

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have $\mu_G(a)$=0.3, $\mu_G(b)$=0.4, $\nu_G(a)$=0.6 and $\nu_G(b)$=0.5. Consider an IFS A=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$. Then by Example 4.9, A^c is an IFωCS but not an IFG″′CS in (X,τ). Hence A is an IFωOS but not an IFG″′OS in (X,τ)

Theorem 4.11 Every IFG″′OS is anIFGOS

Proof Let A be anIFG″′OS in (X,τ). Therefore A^c is an IFG″′CS in (X,τ). Then by Theorem, A^c is an IFGCS in(X,τ). Therefore A is an IFGOS in(X,τ).

The converse of Theorem 4.11 need not be true as seen from the following Example.

Example 4.12

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.6,0.5),(0.3,0.4)\rangle$. We have $\mu_G(a)$=0.6, $\mu_G(b)$=0.5, $\nu_G(a)$=0.3 and $\nu_G(b)$=0.4. Consider an IFS A=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$. Then by Example 4.11, A^c is an IFGCS but not an IFG″′CS in (X,τ). Hence A is an IFGOS but not an IFG″′OS in (X,τ)

Theorem 4.13 Every IFG″′OS is anIFαGOS

Proof Let A be anIFG″′OS in (X,τ). Therefore A^c is an IFG″′CS in (X,τ). Then by Theorem,A^c is an IFαGCS in(X,τ). Therefore A is an IFαGOS in(X,τ).

The converse of theorem 4.13 need not be true as seen from the following Example.

Example 4.14

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.6,0.7),(0.3,0.2)\rangle$. We have $\mu_G(a)$=0.6, $\mu_G(b)$=0.7, $\nu_G(a)$=0.3 and $\nu_G(b)$=0.2. Consider an IFS A=$\langle x,(0.3,0.2),(0.6,0.7)\rangle$. Then by Example 4.13, A^c is an IFαGCS but not an IFG″′CS in (X,τ). Hence A is an IFαGOS but not an IFG″′OS in (X,τ).

Theorem 4 .15 EveryIFG′′′OS is anIFGSOS.

Proof Let A be anIFG′′′OS in (X,τ). Therefore A^c is an IFG′′′CS in (X,τ). Then by Theorem

,A^c is an IFGSCS in(X,τ). Therefore A is an IFGSOS in(X,τ).

The converse of Theorem 4.15 need not be true as seen from the following Example.

Example 4.16

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A=$\langle x,(0.3,0.2),(0.6,0.7)\rangle$. Then by Example, A^c is an IFGSCS but not an IFG′′′CS in (X,τ). Hence A is an IFGSOS but not an IFG′′′OS in (X,τ).

Theorem 4.17 Every IFG′′′OS is anIFG_s'''OS.

Proof Let A be an IFG′′′OS in (X,τ). Therefore A^c is an IFG′′′CS in (X,τ). Then by Theorem, A^c is an IFG_s'''CS in(X,τ).Therefore A is an IFG_s'''OS in(X,τ).

The converse of Theorem 4.17 need not be true as seen from the following Example.

Example 4.18

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.7 and ν_G(b)=0.6. Consider an IFS A=$\langle x,(0.6,0.5),(0.3,0.4)\rangle$. Then by Example 4.18, A^c is an IFG_s'''CS but not an IFG′′′CS in (X,τ). Hence A is an IFG_s'''OS but not an IFG′′′OS in (X,τ).

Theorem 4.19 Every IFG′′′OS is anIFαGSOS.

Proof Let A be anIFG′′′OS in (X,τ). Therefore A^c is an IFG′′′CS in (X,τ). Then by Theorem, A^c is an IFαGSCS in(X,τ).Therefore A is an IFαGSOS in(X,τ).

The converse of Theorem 4.19 need not be true as seen from the following Example.

Example 4.20

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.4,0.3),(0.5,0.6)\rangle$. We have μ_G(a)=0.4, μ_G(b)=0.3, ν_G(a)=0.5 and ν_G(b)=0.6. Consider an IFS A=$\langle x,(0.3,0.2),(0.6,0.7)\rangle$. Then by Example 4.20, A^c is an IFαGSCS but not an IFG′′′CS in (X,τ). Hence A is an IFαGSOS but not an IFG′′′OS in (X,τ).

Theorem 4.21 Every IFG′′′OS is anIFG_α'''OS.

Proof Let A be anIFG″′OS in (X,τ). Therefore A^c is an IFG″′CS in (X,τ). Then by Theorem ,A^c is an IF$G_\alpha^{\prime\prime\prime}$CS in(X,τ).Therefore A is an IF$G_\alpha^{\prime\prime\prime}$OS in(X,τ).

The converse of Theorem 4.21 need not be true as seen from the following Example.

Example 4.22

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.3,0.6),(0.6,0.3)\rangle$. We have μ_G(a)=0.3, μ_G(b)=0.6, ν_G(a)=0.6 and ν_G(b)=0.3. Consider an IFS A=$\langle x,(0.7,0.6),(0.2,0.3)\rangle$. Then A is an IF$G_\alpha^{\prime\prime\prime}$OS but not an IFG″′OS in (X,τ).

Remark 4.23 IFαOS and IFG″′OS are independent.

Example 4.24

Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.2,0.3),(0.8,0.7)\rangle$. We have μ_G(a)=0.2, μ_G(b)=0.3, ν_G(a)=0.8 and ν_G(b)=0.7. Consider an IFS A=$\langle x,(0.9,0.6),(0.1,0.4)\rangle$. Then A is an IF$\alpha$OS but not an IFG″′OS in (X,τ).

Example 4.25

Let X={a,b}. Let τ ={0~,G_1,G_2,1~} be an IFT on X, where G_1=$\langle x,(0.3,0.2),(0.6,0.7)\rangle$ and G_2= $\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have μ_{G1}(a)=0.3, μ_{G1}(b)=0.2, ν_{G1}(a)=0.6, ν_{G1}(b)=0.7, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider the two IFS A=$\langle x,(0.25,0.15),(0.65,0.75)\rangle$. Then A is an IFG″′OS but not an IFαOSin (X,τ).

Remark 4.26 IFSOS and IFG″′OS are independent.

Example 4.27 Let X={a,b}. Let τ ={0~,G,1~} be an IFT on X, where G=$\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have μ_G(a)=0.3, μ_G(b)=0.4, ν_G(a)=0.6 and ν_G(b)=0.5. Consider an IFS A=$\langle x,(0.5,0.4),(0.4,0.5)\rangle$. Then A is an IFSOS but not an IFG″′OS in (X,τ).

Example 4.28 Let X={a,b}. Let τ ={0~,G_1,G_2,1~} be an IFT on X, where G_1=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$ and G_2= $\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have μ_{G1}(a)=0.2, μ_{G1}(b)=0.3, ν_{G1}(a)=0.7, ν_{G1}(b)=0.6, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider the two IFS A=$\langle x,(0.1,0.2),(0.8,0.7)\rangle$. Then A is an IFG″′OS but not an IFSOSin (X,τ).

Theorem 4.29 If A and B are IFG″′OSs in an IFTS (X,τ), then A∩B is also an IFG″′OS in (X,τ).

Proof Let A and B be IFG″′OSs in (X,τ). Therefore A^c and B^c are IFG″′CSs in (X,τ). By theorem , ($A^c \cup B^c$) is an IFG″′CS in (X,τ). Since ($A^c \cup B^c$)=$(A \cap B)^c$, A∩B is also an IFG″′OS in (X,τ).

Remark 4.30 The union of twoIFG″′OSs in an IFTS (X,τ) need not be an IFG″′OS in (X,τ).

Example 4.31 Let X={a,b}. Let τ ={0~,G_1,G_2,1~} be an IFT on X, where G_1=$\langle x,(0.2,0.3),(0.7,0.6)\rangle$ and G_2= $\langle x,(0.3,0.4),(0.6,0.5)\rangle$. We have μ_{G1}(a)=0.2, μ_{G1}(b)=0.3, ν_{G1}(a)=0.7, ν_{G1}(b)=0.6, μ_{G2}(a)=0.3, μ_{G2}(b)=0.4, ν_{G2}(a)=0.6 and ν_{G2}(b)=0.5.Consider the two IFS A= $\langle x,(0.1,0.7),(0.8,0.2)\rangle$ and

B=$\langle x,(0.8,0.2),(0.1,0.7)\rangle$Then A and B are IFG″′Oss. But A∪B=$\langle x,(0.8,0.7),(0.1,0.2)\rangle$is not an IFG″′OS in (X,τ).

Theorem 4.32 If A is an IFG″′OS in an IFTS (X,τ) such that int(A)⊆B⊆A, then B is IFG″′OS in (X,τ).

Proof Let A be an IFG″′OS in(X,τ) such that int(A)⊆B⊆A. It implies$A^c \subseteq B^c \subseteq$ cl(A^c) Where $A^c$is an IFG″′CS in(X,τ). By Theorem, B^c is an IFG″′CS in(X,τ). Therefore B is an IFG″′OS in(X,τ).

Theorem 4 .33 Let (X,τ) be an IFTS. Then IFO(X)=IFG″′O(X) if every IFS in(X,τ) is an IFGSOS in X, where IFO(X) denotes the collection of IFOSs of an IFTS(X,τ).

Proof Suppose that every IFS in(X,τ) is an IFGSOS in X. Then by theorem, we have IFC(X)= IFG″′C(X). Therefore IFO(X)=IFG″′O(X).

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