# ON PRE -$\gamma$-m-$I$-OPEN SETS IN IDEAL MINIMAL SPACES

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**ABSTRACT**

During this paper, we pioneer the notion of pre-$\gamma$-m-$I$ open sets in ideal minimal space. Also, we investigate some properties and characterizations of these sets with suitable examples are given.

## 1. INTRODUCTION

In 1992, Jankovic and Hamlett introduced the notion of I-open sets in topological spaces via ideals. Dontchevin 1999 introduced pre-I-open sets; Kasaharain 1979 defined an operation $\alpha$ on a topological space to introduce $\alpha$-closed graphs. Following the same technique Ogata in 1991 defined an operation $\gamma$ on topological space and introduced $\gamma$-open sets. During this paper, we pioneer the notions of pre-$\gamma$-m-$I$ open sets in ideal minimal space. Also, we investigate some properties and characterizations of these sets with suitable examples are given.

## 2. PRELIMINARIES

**Definition 2.1. [10]** Let $(X,\tau)$ be a topological space and $A$ subset of X. A subset $A$ of a space $(X,\tau)$ is said to be regular open if $A = Int(Cl(A))$ and $A$ is called $\delta$- open if for each $x \in A$ there exist a regular open set $G$ such that $x \in G \subseteq A$.

**Definition 2.2. [7]** A topological space $(X,\tau)$ with an operation $\gamma$ on $\tau$ is said to be $\gamma$-regular if for each $x \in X$ and for each open neighborhood $V$ of $x$ there exist an open neighborhood $U$ of $x$ such that $\gamma(U)$ contained in $V$.

**Definition 2.3. [2, 6]** An ideal is defined as a nonempty collection $I$ of subsets $X$ satisfying the following two conditions:

(i) If $A \in I$ and $B \subseteq A$, then $B \in I$.

(ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

For an ideal $I$ on $(X,\tau)$, $(X,\tau,I)$ is called an ideal topological space or simply an ideal space. Given a topological space $(X,\tau)$ with an ideal $I$ on $X$ and if $P(X)$ is the set of all subsets of $X$, a set operator $(.)^* : P(X) \to P(X)$ called a local function [2], [6] of $A$ with respect to $\tau$ and $I$ is defined as follows for a subset $A$ of $X$, $A^*(I,\tau) = \{x \in X : U \cap A \notin I \ for\ each\ neighborhood\ U\ of x\}$. A kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I,\tau)$, called the $*$-topology, finer than $\tau$, is defined by $Cl^*(A) = A \cup A^*(I,\tau)$ [4]. We will simply write $A^*$ for $A^*(I,\tau)$ and $\tau^*$ for $\tau^*(I,\tau)$.

**Definition 2.4. [8]** Let $(X, m_X)$ be a minimal space with an ideal I on X and $U_m(x) = \{U_m : x \in U_m, U_m \in m_x\}$ be the family of m-open sets which contain x. Also let $(\bullet)^*_m$ be a set operator from P(X) to P(X). For a subset $A \subset X$, $A^*_m(I, m_x) = \{x \in X : U_m \cap A \notin I; \text{for every } U_m \in U_m(x)\}$ is called the minimal local function of A with respect to I and $m_x$. We will simply write $A^*_m$ for $A^*_m(I, m_x)$.

**Definition 2.5. [8]** Let $(X, m_X)$ be a minimal space with an ideal I on X. The set operator m-$Cl^*$ is called a minimal $*$-closure and is defined as m-$Cl^*(A) = A \cup A^*_m$ for $A \subset X$. We will denote by $m^*_x(I, m_x)$ the minimal structure generated by m-$Cl^*$, that is, $m^*_x(I, m_x)$

$=\{U \subset X$: m-$Cl^*(X - U) = X - U\}$. $m_x^*(I, m_x)$ is called $*$-minimal structure which is finer than $m_x$. The elements of $m_x^*(I, m_x)$ are called minimal $*$-open (briefly, m$*$-open) and the complement of an m$*$-open set is called minimal $*$-closed (briefly, m$*$-closed).

If I is an ideal on $(X, m_X)$, then $(X, m_X, I)$ is called an ideal minimal space or ideal m-space.

**Definition 2.6. [9]** A subset A of an ideal minimal space $(X, m_X, I)$ is said to be

(a) m$*$-dense in itself if $A \subset A_m^*$

(b) m$*$-closed if $A_m^* \subset A$.

(c) m$*$-perfect if $A = A_m^*$.

**Definition 2.7.** A subset $A$ of an ideal topological space $(X, m_X, I)$ is said to be

(a) m-preopen [11] if $A \subseteq m_X$ - $Int(m_X$- $Cl(A))$.

(b) m-$I$-open [11] if $A \subseteq m_X$ - $Int(A_m^*)$

(c) $R$-m- $I$-open [9] if $A = m_X$- $Int(m_X$-$Cl^*(A))$.

(d) Pre-m-$I$ –open [9, 11] if $A \subseteq m_X$-$Int(m_X$-$Cl^*(A))$.

(e) Semi-m-$I$-open [9] if $A \subseteq m_X$-$Int(m_X$-$Cl^*(A))$.

(f) $\alpha$-m-$I$-open [11] if $A \subseteq m_X$-$Int(m_X$-$Cl^*(m_X$-$Int(A)))$.

(g) $b$-m-$I$-open if $A \subseteq m_X$-$Int(m_X$-$Cl^*(A) \cup m_X$-$Cl^*(m_X$-$Int(A))$.

(h) Weakly m-$I$-local closed [11] if $A = U \cap K$, where $U$ is an m-open set and $K$ is a $m$ $*$- closed set in $X$.

(i) Locally m-closed [11] if $A = U \cap K$, where $U$ is an m-open set and $K$ is a m-closed set in $X$.

**Lemma 2.8. [11]** A subset $V$ of an ideal space $(X, m_X, I)$ is a weakly m-$I$-local closed set if and only if there exist $K \in m_X$ such that$V = K \cap m_X$-$Cl^*(V)$.

**Definition 2.9. [9]** An ideal topological space $(X, m_X, I)$ is said to be $m$ $*$-externally disconnected if the $m*$-closure of every m-open subset $V$ of $X$ is m-open.

**Theorem 2.10. [9]** For an ideal topological space $(X, m_X, I)$ the following properties are equivalent:

(1) $X$ is $m$ $*$-externally disconnected .

(2) $m_X$-$Cl^*(m_X$-$Int(V)) \subseteq m_X$-$Int(m_X$-$Cl^*(V))$ for every subset $V$of $X$.

**Lemma 2.11. [9, 11]** Let $(X, m_X, I)$ be an ideal topological space and $A, B$ subsets of $X$.then

(1) If $A \subseteq B$, then $A_m{}^* \subseteq B_m{}^*$.

(2) If $U \in m_X$, then $U \cap A_m{}^* \subseteq (U \cap A)_m{}^*$.

(3) $A^*$ is m-closed in $(X, m_X)$.

## 3. Pre - $\gamma$-m-$I$-open sets

**Definition 3.1.** An operation $\gamma$ on a topology $\tau$ is a mapping from $m_X$ in to power set $P(X)$ of $X$ such that $V \subseteq \gamma(V)$ for each $V \in m_X$, where $\gamma(V)$ denotes the value of $\gamma$ at $V$. A subset $A$ of $X$ with an operation $\gamma$ on $m_X$ is called $m$-$\gamma$-open if for each $x \in A$, there exists an m-open set $U$ such that $x \in U$ and $\gamma(U) \subseteq A$.

**Definition 3.2.** In a space $(X, m_X, I)$, $m_{X_\gamma}$ denotes the set of all $m$-$\gamma$-open set in X and complements of $m$-$\gamma$-open sets are called $m$-$\gamma$-closed. The $m_{X_\gamma}$-interior of the $A$ is denoted by $m_{X_\gamma}$-$Int(A)$ and defined to be the union of all $m$-$\gamma$-open sets of X contained in $A$. The $m_{X_\gamma}$-closure of $A$ is denoted by $m_{X_\gamma}$-$Cl(A)$ and defined to be the intersection of all $m$-$\gamma$-closed sets containing$A$.

**Definition 3.3.** A subset $A$ of space $(X, m_X, I)$ is called

(a) Pre- $m$-$\gamma$ -open if $A \subseteq m_{X_\gamma}$-$Int(m_X$- $Cl(A))$.

(b) m-$\gamma$-preopen if $A \subseteq m_{X_\gamma}$- $Int(m_{X_\gamma}$-$Cl(A))$.

(c) m-$\gamma$-p-open if $A \subseteq m_X$-$Int(m_{X_\gamma}$-$Cl(A))$.

Complement of the above sets are closed.

**Proposition 3.4.** Let $\gamma: m_X \rightarrow P(X)$ be a regular operation on $m_X$. If $A$ and $B$ are $\gamma$-m-open , then $A$ ∩B is $\gamma$- m-open.

**Definition 3.5.** A subset $A$ of space $(X, m_X, I)$ with an operation $\gamma$ on $m_X$ is called pre $\gamma$-$m$-$I$-open if $A \subseteq m_{X_\gamma}$-$Int(m_X$-$Cl^*(A))$.

We indicate via $P_\gamma mIO(X, m_X, I)$ the family of all pre $\gamma$-m-$I$-open subsets of $(X, m_X, I)$ or plainly put in writing $P_\gamma mIO(X, m_X, I)\ or P_\gamma mIO(X)$ when there is no chance for confusion with the ideal.

**Theorem 3.6.** Each $\gamma$-m-open set is pre-$\gamma$-$m$- $I$-open in a space $(X, m_X, I)$.

**Proof.**

Let $(X, m_X, I)$ be a minimal space and $A$ a $\gamma$-m-open set of $X$. Then $A = m_{X_\gamma}$-$Int(A) \subseteq m_{X_\gamma}$- $Int(A \cup {A_m}^*) = m_{X_\gamma}$-$Int(m_X$-$Cl^*(A))$.

Opposite is not true exposed in the next example.

**Example 3.7.** Let $X = \{a, b, c\}$ with $m_X = \{\emptyset, X, \{a, c\}\}$ and $I = \{\emptyset, \{b\}\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $A \in m_X$. Then $A = \{b, c\}$ is a pre $\gamma$- $m$-$I$-open set which is not $\gamma$-m-open.

**Theorem 3.8.** Each pre-$\gamma$-$m$-$I$-open set is m-pre-$\gamma$-open in a space $(X, m_X, I)$.

**Proof.**

Let $(X, m_X, I)$ be an minimal space and $A$ be a pre- $\gamma$-$m$-$I$-open set of $X$. Then $A \subseteq m_{X_\gamma}$-$Int(Cl^*(A)) \subseteq m_{X_\gamma}$-$Int(m_X$-$Cl(A))$.

Opposite is not true exposed in the next example.

**Example 3.9.** Let $X = \{a, b, c\}$ with $m_X = \{\emptyset, X, \{a, c\}\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $A \in m_X$. Then $\{a\}$ is a m-pre $\gamma$-open set but not pre- $\gamma$- $m$-$I$-open.

**Theorem 3.10.** Each pre-$\gamma$-$m$-$I$-open set is pre-$m$-$I$-open in a space $(X, m_X, I)$.

**Proof.**

Let $(X, m_X, I)$ be an ideal be a minimal space and $A$ a pre -$\gamma$- $I$-open of $X$.Then $A \subseteq m_{X_\gamma}$- $Int(m_X$-$Cl^*(A)) \subseteq m_X$-$Int(m_X$-$Cl^*(A))$.

Opposite is not true exposed in the next example.

**Example 3.11.** Let $X = \{a, b, c\}$ with $m_X = \{\emptyset, \{c\}\}$ and $I = \{\emptyset, \{c\}\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $A \in m_X$. Then $A = \{c\}$ is a pre $m$-$I$-open set which is not pre-$\gamma$-$m$-$I$-open.

**Theorem 3.12.** Each pre -$\gamma$-$m$- $I$-open set is $m$-$\gamma$- preopen in a space $(X, m_X, I)$.

**Proof.** Let $(X, m_X, I)$ be an ideal minimal space and $A$ a pre -$\gamma$- $m$-$I$-open set of $X$. Then $A \subseteq m_{X_\gamma}$- $Int(m_X\text{-}Cl^*(A)) \subseteq m_{X_\gamma}\text{-}Int(m_X\text{-}Cl(A)) \subseteq m_{X_\gamma}\text{-}Int.$ $(m_{X_\gamma}\text{-}Cl\ (A))$.

Opposite is not true exposed in the next example.

**Example 3.13.** Let$X = \{a, b, c\}$ with $m_X = \{\emptyset, X, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{b\}\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $A \in m_X$ .Then $A = \{b, c\}$ is a $m$-$\gamma$-preopen set which is not pre -$\gamma$- $m$-$I$-open.

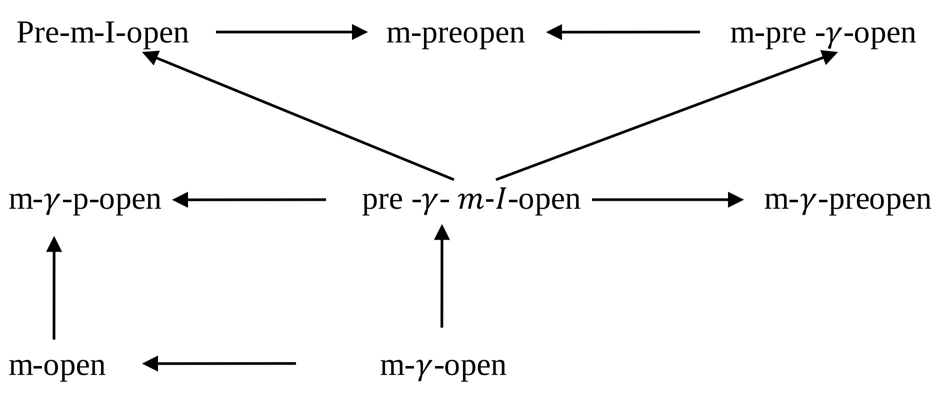
**Theorem 3.14.** Each pre -$\gamma$- $m$-$I$-open set is $m$-$\gamma$-p- open in a space $(X, m_X, I)$.

**Proof.** Let $(X, m_X, I)$ be an ideal minimal space and $A$ a pre -$\gamma$-$m$-$I$-open set of $X$.Then,$A \subseteq m_{X_\gamma}$- $Int(m_X\text{-}Cl^*(A)) \subseteq m_{X_\gamma}\text{-}IntCl(A)) \subseteq Int(m_{X_\gamma}\text{-}Cl\ (A))$.

Opposite is not true exposed in the next example.

**Example 3.15.** Let $X = \{a, b, c\}$ with $m_X = P(X)$ and $I = \{\emptyset\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $A \in m_X$. Then $A = \{c, d\}$ is a m-$\gamma$-p-open set which is not pre -$\gamma$- $m$-$I$-open.

**Remark 3.16.** Above the statements are shown in a Diagram.




None of the above arrows are reversible.

**Proposition. 3.18.** If $A$ and $B$ are pre-$\gamma$- $m$-$I$-open then $A \cap B$ $not\ in$ $P_\gamma mIO(X, m_X)$. Exposed in the next example.

**Example 3.17**. Let $X = \{a, b, c\}$ with $m_X = \{\emptyset, X, \{a, c\}\}$ and $I = \{\emptyset, \{b\}\}$. Define an operation $\gamma$on $m_X$ by $\gamma(A) = X$ for all $\in m_X$ . Set $A = \{a, b\}$ $and$ $B = \{b, c\}$ . Since ${A_m}^* = {B_m}^* = X$, then both $A$ and $B$ are pre-$\gamma$- $m$-$I$-open. But on the other hand $A \cap B = \{b\} \notin P_\gamma mIO(X, m_X)$.

**Theorem 3.18.** Let $(X,\ m_X,\ I)$ be an ideal minimal space and $\{A_\alpha : \alpha \epsilon \Delta\ \}$a family of subsets of $X$ , where $\Delta$ is an arbitrary index set. Then

(i) if $A_\alpha\ \epsilon P_\gamma mIO(X, m_X)$ for all $\alpha\ \epsilon\Delta$, then $\cup_{\alpha\ \epsilon\Delta} A_\alpha\ \epsilon P_\gamma mIO(X,\ m_X)$.

(ii) if $A\ \epsilon P_\gamma mIO(X, m_X)$ and $U\ \epsilon\ m_{X_\gamma}$, then $A \cap U\ \epsilon P_\gamma mIO(X,\ m_X)$. Where $\gamma$is regular operation on $m_X$.

**Proof.** (i). Since $\{A_\alpha : \alpha\ \epsilon\ \Delta\ \} \subseteq P_\gamma mIO(X, m_X)$, then $A_\alpha \subseteq m_{X_\gamma}$-$Int$ $(m_X$-$Cl^*$ $(A_\alpha))$ for each $\alpha\ \epsilon\Delta$. Then we have $\cup_{\alpha\ \epsilon\Delta} A_\alpha \subseteq \cup_{\alpha\ \epsilon\Delta} m_{X_\gamma}$- $Int\left(m_X\text{-}\ Cl^*(A_\alpha)\right)$ $m_{X_\gamma}$-$Int(\cup_{\alpha\ \epsilon\Delta} (m_X$-$Cl^*(A_\alpha))) \subseteq m_{X_\gamma}$-$Int\big(m_X$-$Cl^*(\cup_{\alpha\ \epsilon\Delta} (A_\alpha)\big)$.

(ii). By the assumption $A \subseteq m_{X_\gamma}$–$Int(m_X$-$Cl^*(A))$ and $U = m_{X_\gamma} - Int(U)$. This using lemma (2.11), we have $A \cap U$ $\subseteq m_{X_\gamma}$-$Int(m_X$-$Cl^*(A)) \cap m_{X_\gamma} - Int(U)$ $= m_{X_\gamma} - Int(m_X$-$Cl^*(A) \cap U = m_{X_\gamma} - Int\big(({A_m}^* \cup A) \cap U\big) = m_{X_\gamma}$-$Int({A_m}^* \cap U) \cup (A \cap U) \subseteq$ $m_X$-$_\gamma Int((AU)^*(A \cap U) = m_{X_\gamma}$-$Int(m_X$-$Cl^*(A \cap U))$. This shows that $(A \cap U) \in$ $P_\gamma mIO(X, m_X)$.

**Proposition 3.19**. For an ideal minimal space $(X,\ m_X,\ I)$ with an operation $\gamma$on $m_X$ and $A \subseteq X$, we have

(i) if $I = \{\emptyset\}$, then $A$ is pre -$\gamma$-$m$- $I$-open if and only if $A$ is m-pre -$\gamma$- open.

(ii) if $= P(X)$ , then $P_\gamma mIO(X) = m_{X_\gamma}$.

**Proof.** 1. By the theorem (3.8) we need to show only sufficiency. Let $I = \{\emptyset\}$, then ${A_m}^* = m_X$-$Cl(A)$ for every subset $A$ of $X$. Let $A$ be m-pre -$\gamma$- $I$-open.

2. Let $= P(X)$ , then $A_m{}^* = \emptyset$ for every subset $A$ of $X$. Let $A$ be any pre -$\gamma$-$m$- $I$-open set, then $A \subseteq m_{X_\gamma}$- $Int(m_X$-$Cl^*(A)) = m_{X_\gamma}$-$Int$ $(A \cup A_m{}^*)$= $(A \cup \emptyset) = m_{X_\gamma}$-$Int$ $(A)$ and hence $A$ is $m$-$\gamma$- open. By theorem (3.6), we obtain $P_\gamma mIO(X, m_X)$= $m_{X_\gamma}$.

**Remarks 3.20**.

(i) If a subset $A$ of a $\gamma$- regular space $(X,\ I,\ m_X)$ is open then $A$ is pre -$\gamma$-$m$ -$I$-open.

(ii) If a subset $A$ of a submaximal space $(X,\ I,\ m_X\ )$ is pre -$\gamma$- $m$-$I$-open then $A$ is m-open.

(iii) If $(X, \tau, m_X)$ is $\gamma$- regular space and $I = P(X)$, then $A$ is pre $\gamma$- $m$- $I$-open if and only if $A$is m-open.

**Remark 3.21.** Let $(X,\ m_X,\ I\ )$ be a a $\gamma$- regular space and $I = P(X)$. Then

(i) if $A$ is $R$- $m$- $I$- open then $A$ is pre -$\gamma$-$m$- $I$-open.

(ii) if $A$ is $\delta_{m-I}$- open then $A$ is pre -$\gamma$-$m$- $I$-open.

(iii) if $A$ is regular open then $A$ is pre- $\gamma$- $m$- $I$-open.

(iv) if $A$ is $\delta$- open then $A$ is pre -$\gamma$-$m$- $I$-open.

**Remarks 3.22.** For an ideal topological space $A \subseteq\ (X,\ m_X,\ I)$ with an operation $\gamma\ on\ m_X$ and $I = P(X)$ we have

(i) if $A$ is pre- $\gamma$-$m$- $I$- open then $A$ is m-open.

(ii) if $A$ is pre- $\gamma$-$m$- $I$- open then $A$ is $\alpha$-$m$- $I$-open.

(iii) if $A$ is pre- $\gamma$-$m$- $I$- open then $A$ is semi -$m$- $I$- open.

**Proposition 3.23.** Let $(X\ , m_X,\ I)$ be an ideal minimal space and A be a subset of $X$. If $A$ is closed and pre- $\gamma$-$m$- $I$- open , then $A$ is $R$- $m$- $I$- open.

**Proof.** Let $A$ be pre $\gamma$-$m$-$I$-open, then we have $A \subseteq m_{X_\gamma}$-$Int(m_X$- $Cl^*(A)) \subseteq m_X$- $Int(m_X$- $Cl^*(A)) \subseteq m_X$- $Int(m_X$- $Cl(A)) \subseteq m_X$- $Cl(A) = A$ and hence $A$ is $R$-$m$- $I$-open.

**Remarks 3.24.** Let $(X,\ m_X,\ I)$ is $\gamma$- regular space. If $A \subseteq\ (X,\ m_X,\ I)$ is $R$- $m$- $I$- open, then $A$ is pre- $\gamma$-$m$- $I$- open.

**Remarks 3.25.** If $A \subseteq\ (X,\ m_X,\ I)$ is $\gamma$- regular space and $I = \{\emptyset\}$. Then

(i) $A$ is pre- $\gamma$-$m$- $I$- open if and only if $A$ is m-prepen.

(ii) $A$ is pre- $\gamma$-$m$- $I$- open if and only if $A$ is m-$\gamma$-preopen.

(iii) $A$ is pre- $\gamma$-$m$- $I$- open if and only if $A$ is m-$\gamma$- $p$-open.

**Proposition 3.26.** Let $(X,\ m_X,\ I)$ be an ideal minimal space and A be a subset of $X$. If $I = \{\emptyset\}$ and $A$ is pre- $\gamma$-$m$- $I$- open, then $A$ is m-$I$-open.

**Proof.**

Let $A$ be pre- $\gamma$-$m$- $I$- open, then we have $A \subseteq m_{X_\gamma}\text{-}Int(m_X\text{-}Cl^*(A)) \subseteq m_{X_\gamma}\text{-}Int\big(m_X\text{-}Cl(A)\big) \subseteq m_{X_\gamma}\text{-}Int({A_m}^*) \subseteq m_{X_\gamma}\text{-}Int({A_m}^*)$ and hence $A$ is $m$- $I$-open.

**Remarks 3.27.** If $(X,\ m_X,\ I)$ is a $\gamma$- regular space and $A$ is $\delta_{m\text{-}I}$- open then $A$ is pre- $\gamma$-$m$- $I$- open.

**Remarks 3.28.** If $(X,\ m_X,\ I)$ is a $\gamma$- regular then $A$ is pre- $\gamma$-$m$- $I$- open if and only if $A$ is pre- $m$- $I$- open.

**Proposition 3.29.** If $A \subseteq (X,\ m_X,\ I)$is $m\,*$-perfect and pre- $\gamma$-$m$- $I$- open, then is m-$\gamma$-open.

**Proof.**

Let $A$ be $m\,*$-perfect, then $A = {A_m}^*$ and $A \subseteq m_{X_\gamma}\text{-}Int(m_X\text{-}Cl^*(A)) = m_{X_\gamma}\text{-}Int\ (A \cup {A_m}^*) = m_{X_\gamma}\text{-}Int(A \cup A) = m_{X_\gamma}\text{-}Int(A)$ and hence $A$ is m-$\gamma$ -open.

**Remarks 3.30.** If $A \subseteq (X,\ m_X,\ I)$ is $m\,*$-perfect and pre- $\gamma$-$m$- $I$- open, then $A$ is m-open.

**Proposition 3.31.** If $A$ is ${m_X}^*$- closed in $(X,\ m_X,\ I)$ and pre- $\gamma$-$m$- $I$- open, then $A$ is $m$- $\gamma$- open.

**Proof.**

Let $A$ be pre- $\gamma$-$m$- $I$- open, then $A \subseteq m_{X_\gamma}\text{-}Int\ (A \cup {A_m}^*)= m_{X_\gamma}\text{-}Int(A)$ and hence $A$ is m- $\gamma$- open.

**Proposition 3.32.** If $A$ is ${m_X}^*$- closed in $(X,\ m_X,\ I)$ and pre- $\gamma$-$m$- $I$- open, then $A$ is m-open.

**Proposition 3.33.** If $A$ is $m\,*$-perfect in $(X,\ m_X,\ I)$ an pre- $\gamma$-$m$- $I$- open, then $A$ is $m$- $I$-open.

**Proof.** Let $A$ be pre-$\gamma$-$m$-$I$- open, then $A \subseteq m_{X_\gamma}$-$Int\,(m_X$-$Cl^*(A))$= $m_{X_\gamma}$-$Int(A \cup {A_m}^*) = m_{X_\gamma}$-$Int\,({A_m}^*) \subseteq m_{X_\gamma}$-$Int\,({A_m}^*)$ and hence $A$ is m-$I$- open.

**Proposition 3.34.** If a subset $A$ is $m*$-dense-in-itself in $(X,\ m_X,\ I)$ and pre-$\gamma$-$m$-$I$-open, then $A$ is $m$-$I$- open.

**Proof.** Let $A$ be pre-$\gamma$-$m$-$I$- open, then $A \subseteq m_{X_\gamma}$-$Int\,(m_X$-$Cl^*(A))$= $m_{X_\gamma}$-$Int(A \cup {A_m}^*) = m_{X_\gamma}$-$Int\,({A_m}^*) \subseteq m_{X_\gamma}$-$Int\,({A_m}^*)$ and hence $A$ is m-$I$- open.

**Proposition 3.35.** If a subset $A$ of a $m*$- extremally disconnected $\gamma$-regular space $(X,\ m_X,\ I)$ is a $\alpha$-$m$-$I$-open then $A$ is pre-$\gamma$-$m$-$I$- open.

**Proof.** Let A be $\alpha$-m-$I$-open then $A \subseteq m_X$-$Int\left(m_X\text{-}Cl^*\left(m_X\text{-}Int(A)\right)\right) \subseteq m_X$-$Cl^*\left(m_X\text{-}Int(A)\right) \subseteq m_X$-$Int\left(m_X\text{-}Cl^*(A)\right) = m_{X_\gamma}Int\left(m_X\text{-}Cl^*(A)\right)$ and hence $A$ is pre-$\gamma$-$m$-$I$- open.

**Proposition 3.36.** If a subset $A$ of a $m*$- extremally disconnected $\gamma$-regular space $(X,\ m_X,\ I)$ is a semi-$m$-$I$-open then $A$ is pre-$\gamma$-$m$-$I$- open.

**Proof.** Let $A$ be semi-$m$-$I$-open then $A \subseteq m_X$-$Int\left(m_X\text{-}Cl^*\left(m_X\text{-}Int(A)\right)\right) \subseteq m_X$-$Cl^*\left(m_X\text{-}Int(A)\right)$ and hence $A$ is pre-$\gamma$-$m$-$I$- open.

**Proposition 3.37.** If a subset $A$ of a $m*$- extremally disconnected $\gamma$-regular space $(X, m_X, I)$ is a $b$-$m$-$I$-open $I = P(X)$ then $A$ is pre-$\gamma$-$m$-$I$- open.

**Proof.** Let $A$ be a pre $b$-$m$-$I$-open,

Then$A \subseteq m_X$-$Int\left(m_X\text{-}Cl^*(A)\right) \cup m_X$-$Cl^*\left(Int(A)\right) \subseteq m_X$-$Int(A \cup {A_m}^*) \cup$ $m_X$-$Cl^*(m_X$-$Int(A)) \subseteq m_X$-$Int(A \cup \emptyset) \cup m_X$-$Cl^*(m_X$-$Int(A)) \subseteq m_X$-$Int(A) \cup$ $m_X$-$Cl^*(m_X$-$Int(A)) \subseteq m_X$-$Int(A) \cup (m_X$-$Int(A) \cup Int({A_m}^*)) \subseteq m_X$-$Int(A) \cup$ $m_X$-$Int(A_m)^* \subseteq m_X$-$Cl^*\left(m_X\text{-}Int(A)\right)$ $\subseteq m_X$-$Int(m_X$-$Cl^*(A) =$ $m_{X_\gamma}$-$Int\left(m_X\text{-}Cl^*(A)\right)$ and hence $A$ is pre-$\gamma$-$m$-$I$- open.

**Theorem 3.38.** Let $(X, m_X, I)$ be a m$*$- extremally disconnected $\gamma$-regular ideal minimal space and $V \subseteq X$, the following properties are equivalent:

(i) $V$ is a m-$\gamma$- open set.

(ii) $V$ is a $\alpha$-$m$-$I$- open set and weakly $m$-$I$- local closed.

(iii) $V$ is a is pre- m-$\gamma$-$m$- $I$- open and weakly $m$-$I$- local closed.

(iv) $V$ is a is pre- $\gamma$-$m$- $I$- open and weakly $m$-$I$- local closed.

(v) $V$ is a is semi- $m$-$I$- open and weakly $m$-$I$- local closed.

(vi) $V$ is b- $m$-$I$- open and weakly $m$-$I$- local closed.

**Proof.** It follows from the fact that every $m$-$\gamma$- open set is m-open and every m-open set is $\alpha$-$m$-$I$- open set and weakly $m$-$I$- local closed.

(2) ⇒(3): It follows from Proposition 3.35

(3) ⇒(4), (4) ⇒(5) and (5) ⇒(6): Obvious.

(6) ⇒(1): Suppose that $V$ is b-$m$-$I$- open set and weakly $m$-$I$- local closed set in $X$.It follows that $V \subseteq m_X\text{-}Cl^*\left(m_X\text{-}Int(V)\right) \cup m_X\text{-}Int(m_X\text{-}Cl^*(V))$. Since $V$ is and weakly $m$-$I$- local closed set, then there exist a m-open set $G$ such that $V = G \cap m_X\text{-}Cl^*(V)$. It follows from Theorem 2.10 that $V \subseteq G \cap m_X\text{-}Cl^*(m_X\text{-}Int(V)) \cup m_X\text{-}Int(m_X\text{-}Cl^*(V)) = (G \cap m_X\text{-}Cl^*(m_X\text{-}Int(V))) \cup (G \cap m_X\text{-}Int(m_X\text{-}Cl^*(V))) \subseteq (G \cap m_X\text{-}Int(m_X\text{-}Cl^*(V))) \cup (G \cap m_X\text{-}Int(m_X\text{-}Cl^*(V)))$

$= m_X\text{-}Int(G \cap m_X\text{-}Cl^*(V)) \cup m_X\text{-}Int(G \cap m_X\text{-}Cl^*(V)) = m_X\text{-}Int(V) \cup m_X\text{-}Int(V)$

$$= m_X\text{-}Int(V) = m_{X_\gamma}\text{-}Int(V)$$

Thus, $V \subseteq m_{X_\gamma}\text{-}Int(V)$ and hence $V$ is a $m$-$\gamma$-open set in $X$.

**Theorem 3.39.** Let $(X,\ m_X,\ I)$ be a $m*$- extrernally disconnected $\gamma$-regular ideal space and $V \subseteq X$, the following properties are equivalent:

(i) $V$ is a m-$\gamma$- open set.

(ii) $V$ is $\alpha$-m-$I$- open and a locally m-closed set.

(iii) $V$ is pre- $\gamma$-$m$- $I$- open and a locally m-closed set.

(iv) $V$ is pre- $m$-- open and a locally m-closed set.

(v) $V$ is semi- $m$-$I$- open and a locally m-closed set.

(vi) $V$ is b-$m$-$I$-open and a locally m-closed set.

**Proof.** By theorem 3.38, It follows from the fact that every m-$\gamma$- open set is m-open and every m-open set is $\alpha$- $m$-$I$- open set and weakly $m$-$I$- local closed.

**Theorem 3.40.** A subset $A$ of a space $(X,\ m_X,\ I)$ is pre-$\gamma$-$m$-$I$-closed if and only if $m_{X_\gamma}$-$Int\ (m_X$-$Cl^*(A)) \subseteq A$.

**Proof.** Let $A$ be a pre-$\gamma$-$m$-$I$- closed set of $(X,\ m_X,\ I)$, Then $X - A$ is pre-$\gamma$-$m$-$I$- open and hence $X - A \subseteq m_{X_\gamma}$-$Int\ \left(m_X\text{-}Cl^*(A)\right) = X - m_{X_\gamma}$-$Cl(m_X$-$Int^*(A))$. Therefore, we have $m_{X_\gamma}$-$Cl(m_X$-$Int^*(A)) \subseteq A$.

Conversely, let $m_{X_\gamma}$-$Cl(m_X$-$Int^*(A)) \subseteq A$. Then $X - A$ is pre-$\gamma$-$m$-$I$- closed.

**Theorem 3.41.** A subset $A$ of a space $(X, m_X, I)$ is pre-$\gamma$-$m$-$I$-closed if and only if $m_{X_\gamma}$-$Int\ (m_X$-$Cl^*(A)) \subseteq A$.

**Proof.** Let $A$ be any pre-$\gamma$-$m$-$I$-closed set of $(X, m_X, I)$ . since $m_X{}^*(I)$ is finer than $m_X$ and $m_X$ is finer than $m_{X_\gamma}$ ,we have $m_X Cl(m_{X_\gamma}$-$Int(A)) \subseteq m_{X_\gamma}$-$Cl(m_{X_\gamma}$-$Int(A)) \subseteq m_{X_\gamma}$-$Cl(m_X$-$Int^*(A))$. Therefore by Theorem 3.41, we obtain $m_X$-$Cl(m_{X_\gamma}$-$Int(A)) \subseteq A$.

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